Chapter Outline

RL = reinforcement learning

Consensus and Formation Control

Chapter 10 Swarm Collaboration and Game-Theoretic Confrontation

10.1 Swarm Fundamentals and Architectures

10.2 Consensus and Formation Control

10.3 Game Theory and RL Fundamentals

10.4 Single-Agent RL Algorithms

10.5 Multi-Agent RL and Scaling

10.6 Technologies and Challenges

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10.2.1 First-Order Consensus and Convergence Conditions

Textbook: Sec. 10.1.2.3

10.2.2 Position Offsets and Dynamic Formations

Textbook: Sec. 10.1.2.3

10.2.3 Leader–Follower and Virtual Structure Methods

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10.2.4 Virtual Tubes and Behavior-Based Methods

Textbook: Sec. 10.1.2.4

10.2.1 First-Order Consensus and Convergence Conditions: Model and Notation

States Available Through the Communication Graph



$$\mathcal{N}_1 = \{2\}, \quad \mathcal{N}_2 = \{1,3\}, \quad \mathcal{N}_3 = \{2\}$$

Neighborhood: $\mathcal{N}_i$ contains the neighbors that node i can communicate with. Each edge in this chain has weight 1.
Prerequisite: Chapter 9 graph theory. Undirected edges are bidirectional. Fixed topology is constant over time.

Assumptions: positive symmetric edge weights, a connected graph, and ideal integrators. No delay, packet loss, saturation, or disturbance.

10.2.1 First-Order Consensus and Convergence Conditions: Local Protocol

First-Order Feedback From Neighbor Differences

$$\mathbf{u}_i = -\sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_i - \mathbf{x}_j) \tag{10.1}$$

$$\dot{\mathbf{x}}_i = \mathbf{u}_i \tag{10.2}$$

$a_{ij} > 0$: control weight between neighbors. $\mathbf{x}_i$: state. $\mathbf{u}_i$: state rate.

Neighbor state differences produce the local command. Contributions from opposite sides can cancel, so inspect their sum.

The feedback sign determines whether errors decrease or grow.

10.2.1 First-Order Consensus and Convergence Conditions: Matrix Form and Consensus Value

Laplacian Form of the Collective Dynamics

$$\dot{\mathbf{x}} = -\mathbf{L}\mathbf{x}, \qquad \mathbf{L} = \mathbf{D} - \mathbf{A} \tag{10.3}$$

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \quad d > 1: \dot{\mathbf{x}} = -(\mathbf{L} \otimes \mathbf{I}_d)\mathbf{x}$$

$$\lim_{t\to\infty} \mathbf{x}_i(t) = \frac{1}{N}\sum_{k=1}^{N} \mathbf{x}_k(0) = \mathbf{c}_0 \tag{10.4}$$

$A = [a_{ij}]$: adjacency matrix. $D = \mathrm{diag}(1,2,1)$: degree matrix of this chain.

N: node count. d: state dimension per node. I_d: identity matrix. $\otimes$: Kronecker product.

c_0: initial mean, for a fixed, undirected, connected graph with positive symmetric weights and ideal continuous dynamics.

10.2.1 First-Order Consensus and Convergence Conditions: Three-Node Example

One Step With a Stationary Middle Node

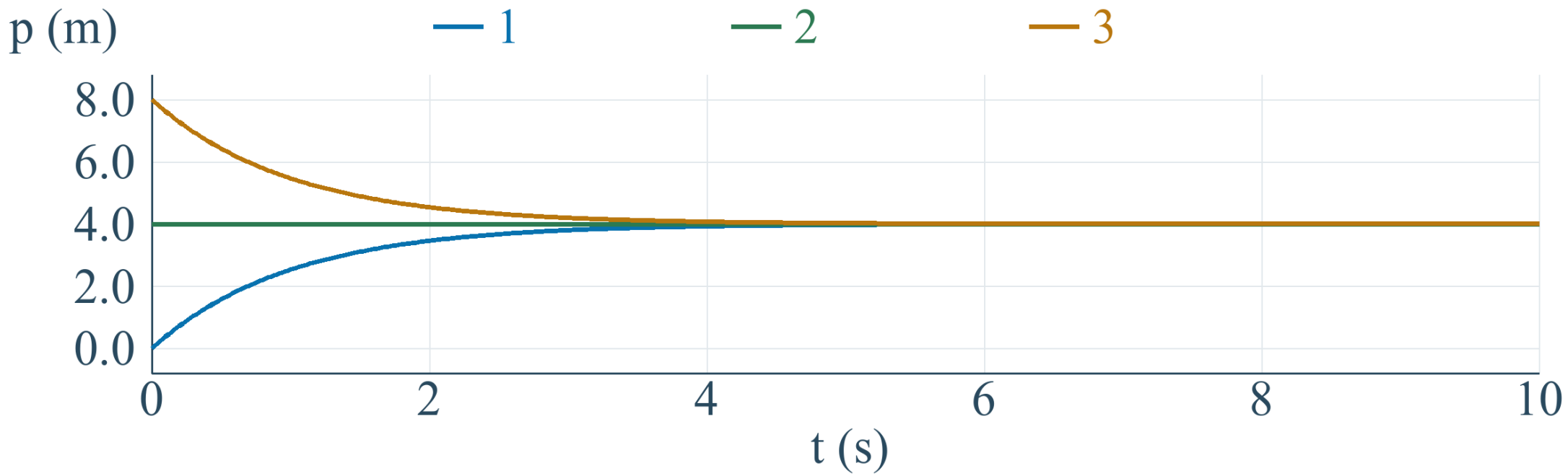
$$\mathbf{p}(0) = [0,4,8]^{\mathsf{T}}, \quad \mathbf{u}(0) = -\mathbf{L}\mathbf{p}(0) = [4,0,-4]^{\mathsf{T}}$$

Node	Local Velocity Calculation (m/s)	Position After 0.2 s (m)
1	$-(0-4)=4$	$0+0.2\times 4=0.8$
2	$-[(4-0)+(4-8)]=0$	4
3	$-(8-4)=-4$	$8-0.2\times 4=7.2$

Explicit Euler example only. Largest graph eigenvalue: 3. Step must be below $2/3$ s.

10.2.1 First-Order Consensus and Convergence Conditions: Analytical Response

Three Virtual Positions Converge to Their Mean



Analytical position-consensus response of the ideal first-order model

$$p_1 = 4 - 4e^{-t}, \quad p_2 = 4, \quad p_3 = 4 + 4e^{-t}$$

10.2.1 First-Order Consensus and Convergence Conditions: Convergence and Limits

Connectivity Gives One Shared Consensus Value

$$V = \frac{1}{2}\|\mathbf{x} - \bar{x}\mathbf{1}\|^2, \qquad \dot{V} = -\mathbf{x}^\top \mathbf{L}\mathbf{x} \leq 0$$

For one coordinate, $\bar{x}$ is the node mean, $\mathbf{1}$ the all-ones vector, and V the disagreement energy.

Topology and Conditions	Property or Result	Conclusion
Fixed, undirected, connected chain	Laplacian eigenvalues: 0, 1, 3	Convergence to the initial mean
Remove the edge between 2 and 3	Final states: 2, 2, 8	Consensus within each component
Delay or topology switching	The current fixed-matrix model no longer applies	Reassess the relevant convergence conditions

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10.2.2 Position Offsets and Dynamic Formations: Position Offsets

Consensus of Shifted States Preserves Spacing

$$\mathbf{v}_i^* = \mathbf{u}_i = -\sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{p}_i - \mathbf{p}_j) \tag{10.5}$$

$$\mathbf{u}_i = \dot{\mathbf{x}}_i = -\sum_{j\in\mathcal{N}_i} a_{ij}[(\mathbf{x}_i - \mathbf{x}_j) - (\boldsymbol{\delta}_i - \boldsymbol{\delta}_j)] \tag{10.6}$$

$$\mathbf{q}_i = \mathbf{x}_i - \boldsymbol{\delta}_i, \quad \mathbf{q}_i \to \overline{\mathbf{q}}(0), \quad \mathbf{x}_i \to \overline{\mathbf{q}}(0) + \boldsymbol{\delta}_i$$

$x_i = p_i$: position. δ_i: fixed world-frame offset. Both use m. Output: m/s.
$\overline{q}(0)$: initial mean of shifted states. The actual position mean remains its initial value.
$\overline{q}(0)$ equals the actual formation center only when the offsets sum to zero.

10.2.2 Position Offsets and Dynamic Formations: Offset Signs

One Step Using the Relative Offset

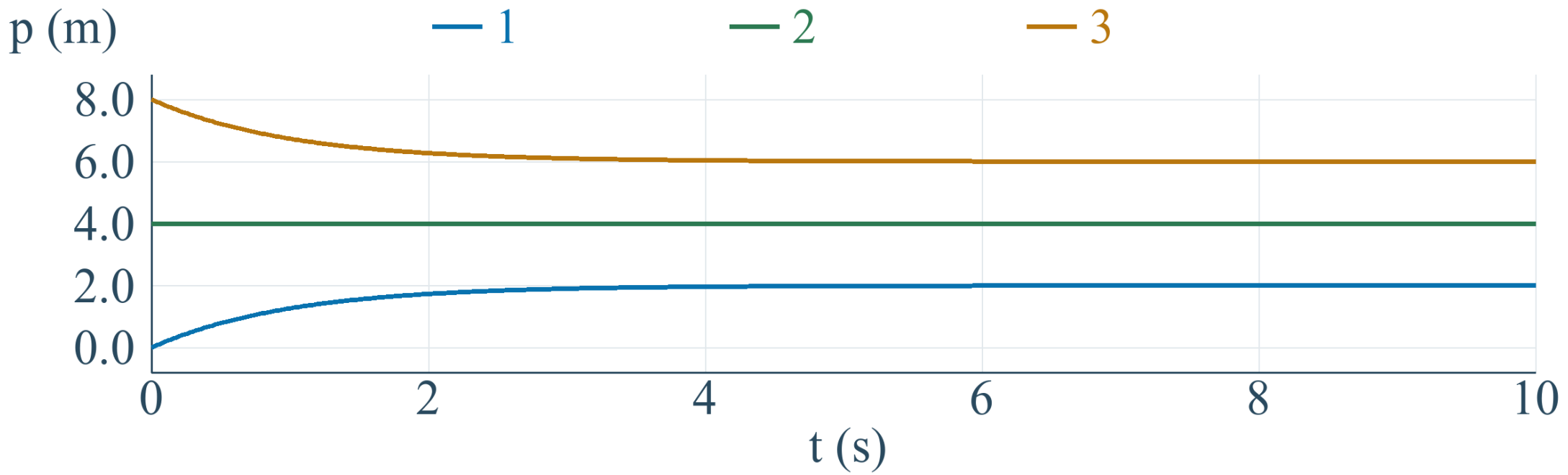
$$\boldsymbol{\delta} = [-2,0,2]^{\mathrm{T}}, \quad \mathbf{q}(0) = \mathbf{p}(0) - \boldsymbol{\delta} = [2,4,6]^{\mathrm{T}}$$

Quantity	One-Step Result for Node 1
Actual relative position	$0 - 4 = -4$ m
Desired relative offset	$-2 - 0 = -2$ m
Correct command	$-[-4 - (-2)] = 2$ m/s
Position after 0.2 s	$0 + 0.2 \times 2 = 0.4$ m

The wrong plus sign gives 6 m/s and fails to give zero even at the desired spacing.

10.2.2 Position Offsets and Dynamic Formations: Offset Response

The Same Initial States Reach 2 m Spacing



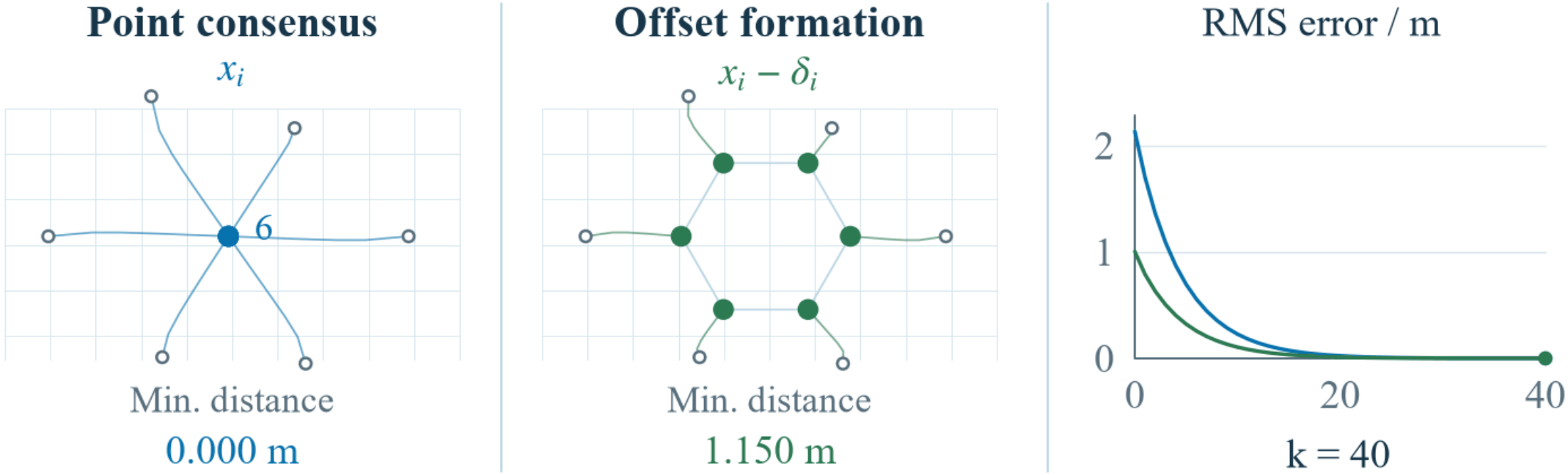
Shifted states reach consensus while actual positions retain spacing

$$p_1 = 2 - 2e^{-t}, \quad p_2 = 4, \quad p_3 = 6 + 2e^{-t}$$

10.2.2 Position Offsets and Dynamic Formations: Six-Node Topology Comparison

Consensus and Offset Formation on a Six-Node Ring

Fixed undirected connected ring. Actual positions plotted, with consensus variables above.



Six-node ring: position consensus and offset formation, separate from the three-node chain.
Root mean square (RMS) error is relative to the consensus value. k is the step index, with a 0.2 s step.

10.2.2 Position Offsets and Dynamic Formations: Dynamic Formation

Second-Order Position and Velocity Feedback

$$\dot{\mathbf{p}}_i = \mathbf{v}_i, \quad \dot{\mathbf{v}}_i = \mathbf{a}_i = \mathbf{u}_i \tag{10.7}$$

$$\mathbf{a}_i = -\sum_{j \in \mathcal{N}_i} a_{ij}[(\mathbf{p}_i - \mathbf{p}_j) - (\boldsymbol{\delta}_i - \boldsymbol{\delta}_j) + \gamma(\mathbf{v}_i - \mathbf{v}_j)] \tag{10.8}$$

p_i, v_i, a_i are position, velocity, and acceleration (m, m/s, $\mathrm{m/s^2}$).
Here a_{ij} includes the position gain ($\mathrm{s^{-2}}$). $\gamma > 0$ is a damping parameter (s).

Conditions: a fixed, undirected, connected graph, positive symmetric weights, and ideal double integrators. No saturation, delay, or disturbance.

10.2.2 Position Offsets and Dynamic Formations: Second-Order Calculation

Velocity Differences Change Acceleration Corrections

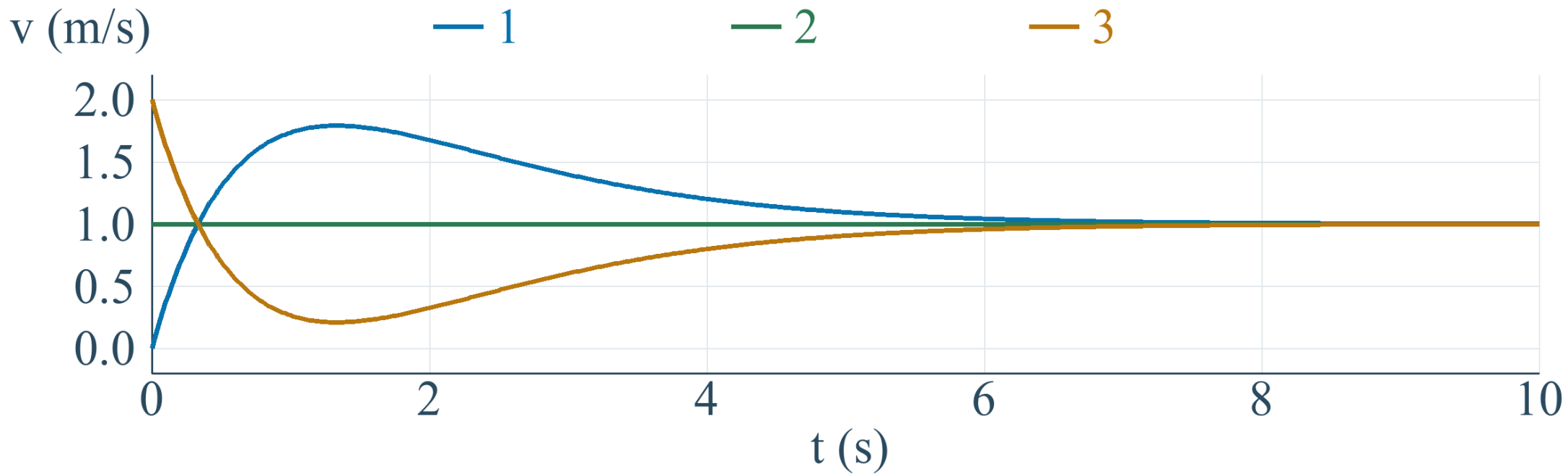
$$\mathbf{p} = [0,4,8]^{\mathsf{T}}, \quad \mathbf{v} = [0,1,2]^{\mathsf{T}}, \quad \gamma = 2$$

Node	Initial Acceleration (m/s²)	Position, 0.1 s (m)	Velocity at 0.1 s (m/s)
1	$-[-2 + 2 \times (-1)] = 4$	0	0.4
2	0	4.1	1.0
3	$-[2 + 2 \times 1] = -4$	8.2	1.6

One explicit Euler step. Complete response: continuous-time analytical solution.

10.2.2 Position Offsets and Dynamic Formations: Second-Order Response

Velocity Consensus at 1 m/s in a Moving Formation



Second-order formation velocity response, with mean velocity fixed at 1 m/s

$$\bar{p} = 4 + t, \quad \mathbf{p} - \bar{p}\mathbf{1} - \boldsymbol{\delta} = [-1,0,1]^{\mathrm{T}}(2 + 3t)e^{-t}$$

10.2.2 Position Offsets and Dynamic Formations: Extensions and Limits

Moving References and Safety Constraints

$$\mathbf{u}_i = \dot{\mathbf{r}} - \sum_{j \in \mathcal{N}_i} a_{ij}[(\mathbf{x}_i - \mathbf{x}_j) - (\boldsymbol{\delta}_i - \boldsymbol{\delta}_j)]$$

Moving center: $r(t)$ is the reference position. $\dot{r}(t)$ is shared velocity feedforward for translation.
The absolute center position still depends on initialization or anchoring.

Safety throughout motion: check minimum distances, obstacles, actuator limits, and how collision-avoidance terms affect stability.

Extensions: event-triggered updates reduce update frequency. Adaptive gains and heterogeneous hierarchies require their own conditions.

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10.2.3 Leader–Follower and Virtual Structure Methods: Structure Overview

References, Spatial Constraints, and Behavior Organization

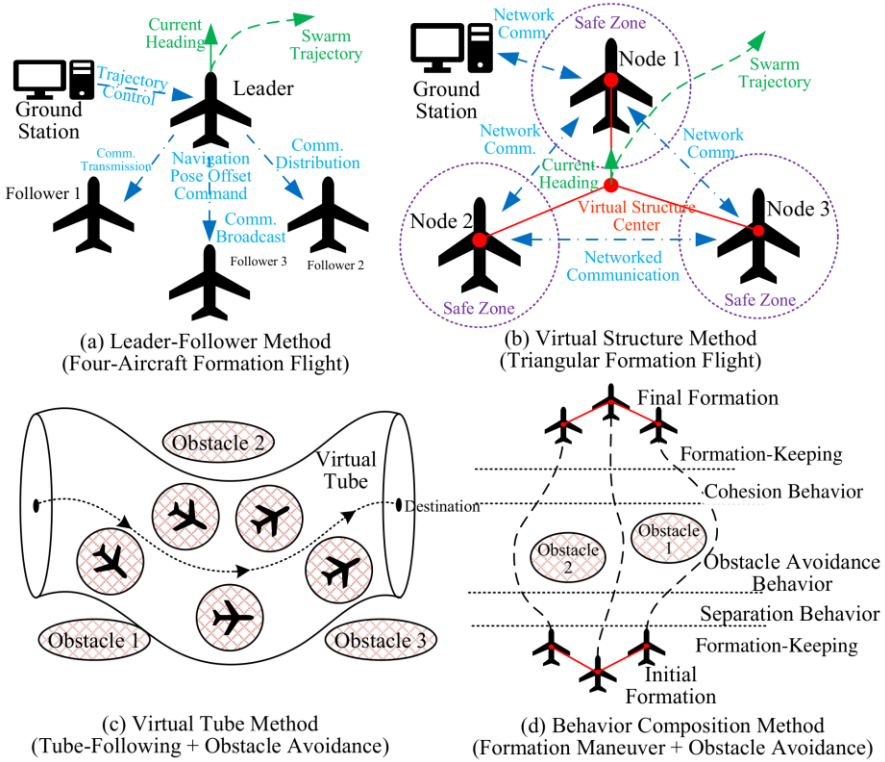


Fig. 10.3 Formation control methods

Leader–follower: reference from a physical leader or a preceding node.

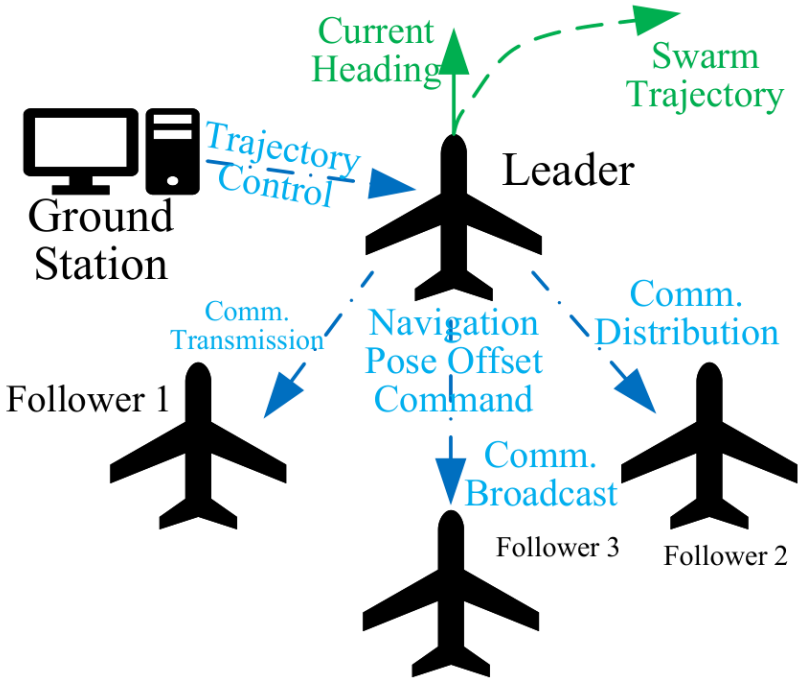
Virtual structure: reference from a common pose and fixed offsets.

Virtual tube: constraints from feasible space and local spacing.

Behavior-based methods: commands from behavior blending or switching.

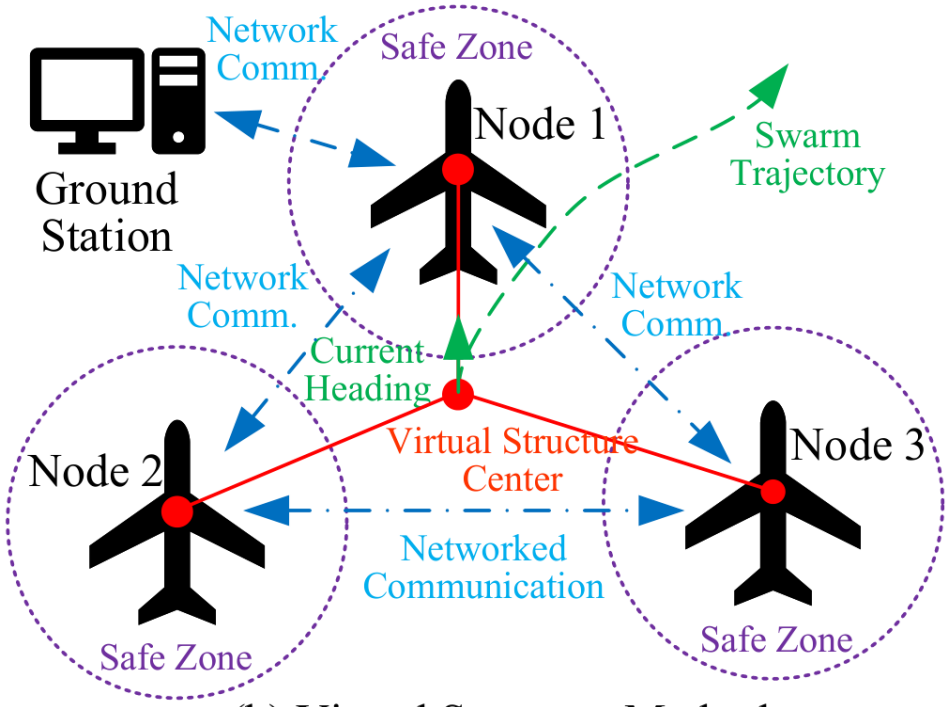
10.2.3 Leader–Follower and Virtual Structure Methods: Pose References

Physical Leaders and Shared Virtual Poses



(a) Leader-Follower Method
(Four-Aircraft Formation Flight)

Fig. 10.3(a) Leader pose and offset commands

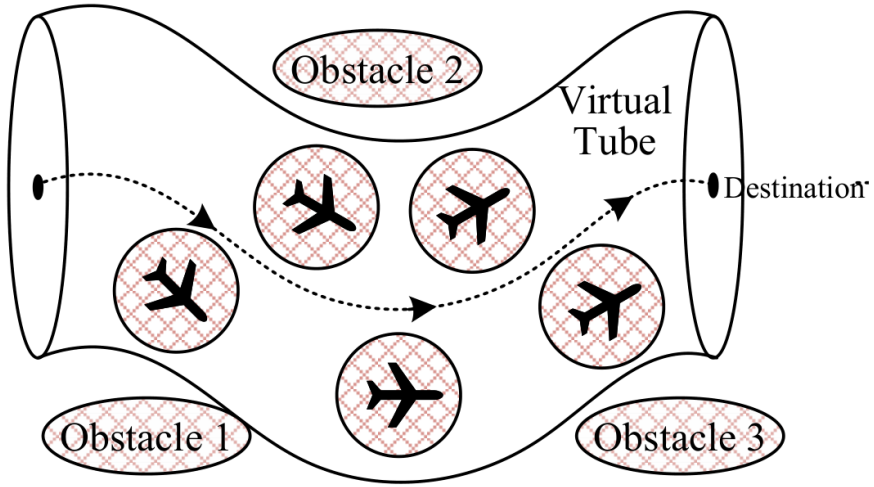


(b) Virtual Structure Method
(Triangular Formation Flight)

Fig. 10.3(b) Member targets from a shared pose

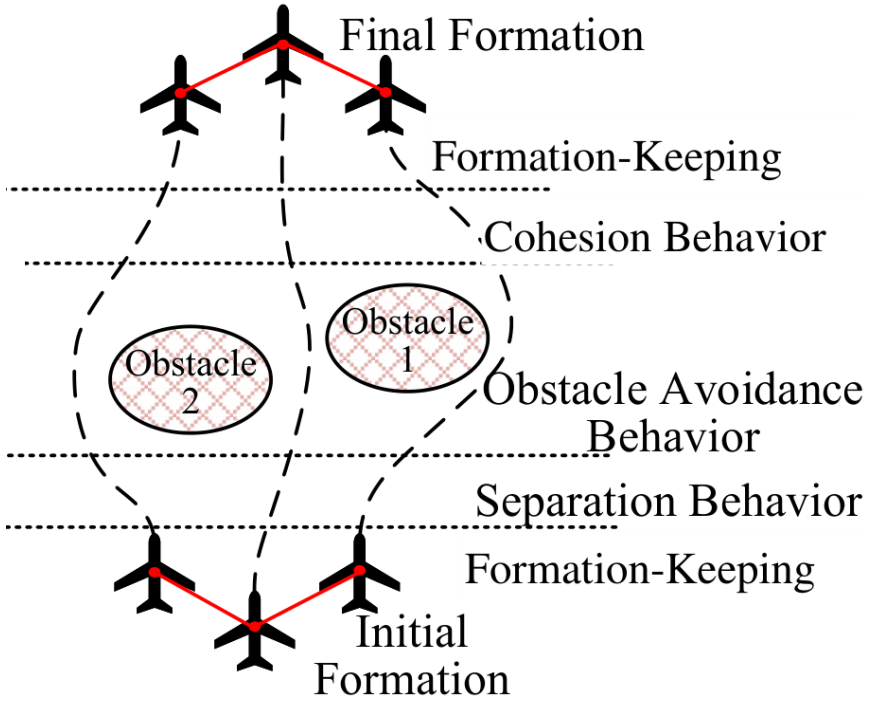
10.2.3 Leader–Follower and Virtual Structure Methods: Space and Behaviors

Feasible Space and Behavior Switching



(c) Virtual Tube Method
(Tube-Following + Obstacle Avoidance)

Fig. 10.3(c) Motion through a tube around obstacles



(d) Behavior Composition Method
Formation Maneuver + Obstacle Avoidance)

Fig. 10.3(d) Behavior switching by phase

10.2.3 Leader–Follower and Virtual Structure Methods: Leader–Follower Control

Leader Position Plus Offset as the Target

$$\mathbf{u}_i = -b_i[\mathbf{x}_i - (\mathbf{x}_{\text{leader}} + \boldsymbol{\delta}_i)] \tag{10.9}$$

Leader Position → **Add Relative Offset** → **Local Negative Feedback**

One step: leader 5 m, offset −2 m, follower 1 m, gain 2 s^{-1}. Target: 3 m. Command: 4 m/s.

Use and risk: straightforward implementation. Leader failure or invalid state data requires backup leaders and reselection.

10.2.3 Leader–Follower and Virtual Structure Methods: Anchoring and Neighbors

Leader Anchoring and Neighbor Offset Feedback

$$\mathbf{u}_i = -b_i[\mathbf{x}_i - (\mathbf{x}_{\text{leader}} + \boldsymbol{\delta}_i)] - \sum_{j\in\mathcal{N}_i} a_{ij}[(\mathbf{x}_i - \mathbf{x}_j) - (\boldsymbol{\delta}_i - \boldsymbol{\delta}_j)] \tag{10.10}$$

$e_i = x_i - (x_{\text{leader}} + \delta_i)$, and e stacks the follower errors.
L_f is the Laplacian of the follower graph.

$$\dot{\mathbf{e}} = -(\mathbf{L}_f + \mathbf{B})\mathbf{e}, \qquad \mathbf{B} = \text{diag}(b_i)$$

Sufficient conditions for a stationary leader: a fixed, undirected, connected follower graph, with at least one follower receiving leader data and using a positive anchoring gain.
The neighborhood excludes the leader. Each disconnected component needs its own valid anchor.

10.2.3 Leader–Follower and Virtual Structure Methods: Moving-Reference Signs

Feedforward With the Same Sign as Leader Velocity

$$\mathbf{e}_i = \mathbf{x}_i - (\mathbf{x}_L + \boldsymbol{\delta}_i), \qquad \dot{\mathbf{e}}_i = \mathbf{u}_i - \dot{\mathbf{x}}_L$$

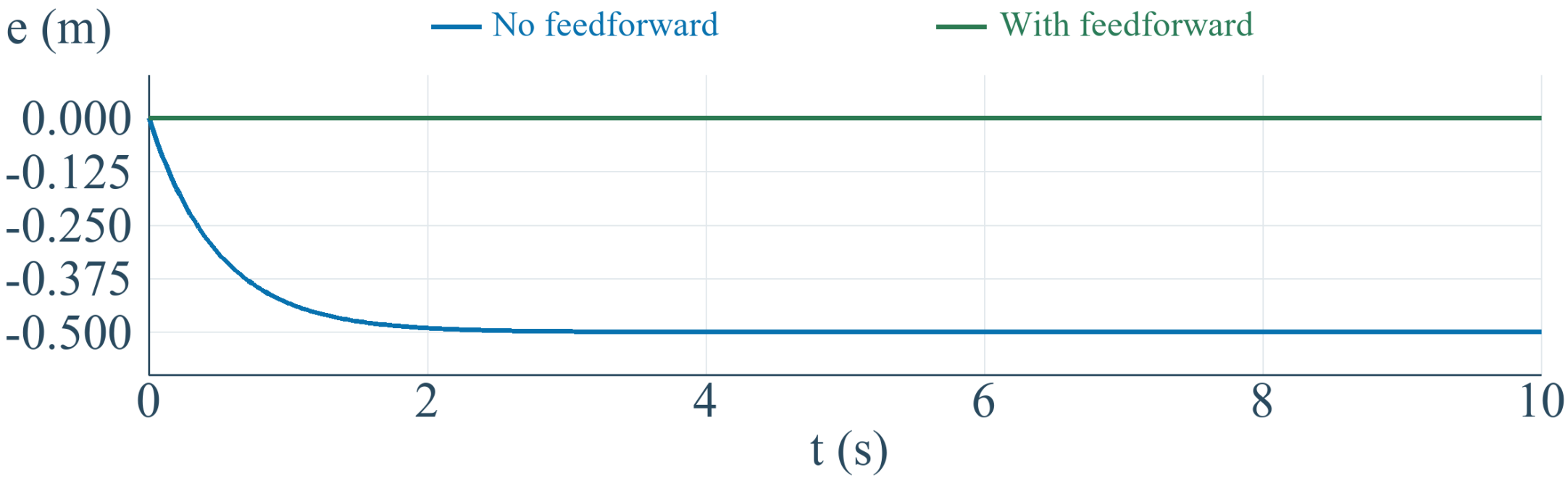
$$\mathbf{u}_i = -b_i \mathbf{e}_i \;\Rightarrow\; \mathbf{e}_{i,ss} = -\dot{\mathbf{x}}_L / b_i$$

$$\mathbf{u}_i = \dot{\mathbf{x}}_L - b_i \mathbf{e}_i \;\Rightarrow\; \dot{\mathbf{e}}_i = -b_i \mathbf{e}_i$$

One-step check: leader velocity 1 m/s, gain 2 s^{-1}, current error −1 m.
Correct feedforward gives 3 m/s. The wrong minus sign gives 1 m/s, so the error initially stays constant.

10.2.3 Leader–Follower and Virtual Structure Methods: Leader-Tracking Response

Formation Keeping Can Coexist With Tracking Error



Tracking error for a constant-speed leader, with and without velocity feedforward

$$v_L = 1,\ b = 2,\ e(0) = 0: \quad e_{\text{no ff}} = -0.5(1 - e^{-2t}), \quad e_{\text{ff}} = 0$$

10.2.3 Leader–Follower and Virtual Structure Methods: Virtual Structure

Member Targets From a Shared Pose

$$(\mathbf{x}_{vs}, \mathbf{R}) \in \mathbb{R}^3 \times SO(3) \tag{10.11}$$

$$\mathbf{x}_i^{des} = \mathbf{x}_{vs} + \mathbf{R}\boldsymbol{\delta}_i \tag{10.12}$$

Position and Rotation → **Rotate Fixed Offsets** → **Member Targets**

$\mathbf{x}_{vs}$: center in the world frame. $\mathbf{R}$: rotation from the structure frame to the world frame.

$\boldsymbol{\delta}_i$: fixed offset in the structure frame. Offsets sum to zero when the center is the centroid.

10.2.3 Leader–Follower and Virtual Structure Methods: Virtual-Structure Errors

Tracking Convergence Recovers the Rigid Geometry

$$\mathbf{e}_i = \mathbf{x}_i - \mathbf{x}_i^{\mathrm{des}} = \mathbf{x}_i - (\mathbf{x}_{\mathrm{vs}} + \mathbf{R}\boldsymbol{\delta}_i) \tag{10.13}$$

$$\mathbf{x}_i - \mathbf{x}_j \to \mathbf{R}(\boldsymbol{\delta}_i - \boldsymbol{\delta}_j), \quad \forall i, j \in \mathcal{V} \tag{10.14}$$

$$\mathbf{u}_i = \dot{\mathbf{x}}_{\mathrm{vs}} + \dot{\mathbf{R}}\boldsymbol{\delta}_i - k\mathbf{e}_i \;\Rightarrow\; \dot{\mathbf{e}}_i = -k\mathbf{e}_i$$

$k > 0$ is the tracking gain (s^{-1}). $\dot{x}_{\mathrm{vs}}$ and $\dot{R}\delta_i$ compensate for translation and rotation. Feedforward needs synchronized clocks and references. Tracking error, communication loss, and saturation disrupt the ideal geometry.

10.2.3 Leader–Follower and Virtual Structure Methods: Rotation Example

Rotate the Offset Before Adding the Center

$$\mathbf{x}_{\mathrm{vs}} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \boldsymbol{\delta}_i = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\mathbf{x}_i^{\mathrm{des}} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix} \ \mathrm{m}$$

$$\mathbf{x}_i = \begin{bmatrix} 8.6 \\ 7.2 \end{bmatrix}, \quad \mathbf{e}_i = \begin{bmatrix} -0.4 \\ 0.2 \end{bmatrix}, \quad -2\mathbf{e}_i = \begin{bmatrix} 0.8 \\ -0.4 \end{bmatrix} \ \mathrm{m/s}$$

Using the unrotated offset gives (12, 6) m, the wrong target for the rotated structure.

10.2.3 Leader–Follower and Virtual Structure Methods: Virtual-Structure Results

Error Decay and Rigid Formation Spacing

$$\mathbf{e}_i(t) = \begin{bmatrix} -0.4 \\ 0.2 \end{bmatrix} e^{-2t}, \qquad \|\mathbf{e}_i(t)\| = \sqrt{0.2}\, e^{-2t}$$

Time (s)	Horizontal Error (m)	Vertical Error (m)	Error Norm (m)
0	−0.4000	0.2000	0.4472
1	−0.0541	0.0271	0.0605
2	−0.0073	0.0037	0.0082
4	−0.00013	0.00007	0.00015

Ideal result. Check clock sync, localization, feedforward, and actuator limits.

10.2.3 Leader–Follower and Virtual Structure Methods: Example Connection

A Shared Trajectory Plus Individual Offsets

Example entry: 10.RflySimSwarm/2.AdvExps/
e1.MatCentCtrlUAVsSim/7.RflyUdpSimpleVirStruct

$$\mathbf{c}(t) = [0.75\cos(0.15t),\ 0.5\sin(0.3t)]^{\mathsf{T}}, \quad \mathbf{p}_i^* = \mathbf{c} + \mathbf{d}_i$$

Node	Example Offset (m)	Target at t = 0 (m)
1	(0, 1)	(0.75, 1)
2	(1, 0)	(1.75, 0)
3	(−1, 0)	(−0.25, 0)

Code substitution only. Offsets use a reference point that need not be the centroid.

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10.2.4 Virtual Tubes and Behavior-Based Methods: Virtual Tubes

Feasible Regions With Flexible Relative Spacing

$$\mathbf{u}_i = \mathbf{u}_{\text{tube},i} + \mathbf{u}_{\text{rep},i} \tag{10.15}$$

Tube Centerline and Forward Direction → **Navigation: forward motion and lateral correction**

Inter-Vehicle and Boundary Distances → **Repulsion: correction of unsafe approach and boundary crossing**

Conditions: a feasible tube, reliable localization, and executable commands. Narrow sections also require congestion and yielding rules.

10.2.4 Virtual Tubes and Behavior-Based Methods: Tube Example

One Euler Step With Forward and Lateral Motion

2D straight tube: x is forward, y lateral, centerline $y = 0$, half-width 2 m.
Ideal velocity execution. No dynamic obstacles.

$$\mathbf{u}_{\text{tube}} = (1, -0.5y), \quad \mathbf{u}_{\text{rep}} = (0, -\max(y - 1, 0)), \quad y \geq 0$$

$$y = 1.5: \quad \mathbf{u} = (1, -0.75) + (0, -0.5) = (1, -1.25)\ \text{m/s}$$

$$\Delta t = 0.2\ \text{s}: \quad (x^+, y^+) = (x + 0.2, 1.25)\ \text{m}$$

Use symmetric correction below the centerline. This example does not establish safety for general potential fields.

10.2.4 Virtual Tubes and Behavior-Based Methods: Tube Results

Piecewise Decay Under Continuous Feedback

$$y(t) = \begin{cases} \frac{2}{3} + \frac{5}{6} e^{-1.5t}, & 0 \le t \le t_c \\ e^{-0.5(t - t_c)}, & t > t_c \end{cases}, \quad t_c = \frac{\ln 2.5}{1.5}$$

Time (s)	Forward Position (m)	Lateral Position (m)
0	0	1.500
1	1	0.823
4	4	0.184
10	10	0.009

t_c is when y reaches 1 m and wall repulsion becomes zero. Narrow sections still need traffic rules.

10.2.4 Virtual Tubes and Behavior-Based Methods: Behavior Modules

Six Basic Behaviors for Swarm Tasks

Aggregation	**Target Tracking**	**Formation Keeping**
Move toward the neighborhood center	Move toward the task target	Maintain relative geometry
Obstacle Avoidance	**Velocity Matching**	**Separation**
Correct unsafe motion	Coordinate motion	Maintain safe spacing

A behavior tree (BT) or finite state machine (FSM) manages calls, priorities, and switching conditions. A geometric controller can serve as one behavior.

10.2.4 Virtual Tubes and Behavior-Based Methods: Combination Rules

Weighted Blending, Arbitration, and Event Switching

$$\mathbf{u}_i = \sum_{k=1}^{m} w_k \mathbf{u}_i^{(k)}, \qquad \sum_{k=1}^{m} w_k = 1 \tag{10.16}$$

Method	Rule	Checks
Weighted blending	Weight and add vectors with the same units	Cancellation, scale differences, and saturation
Priority arbitration	Select a high-priority behavior when its condition triggers	Trigger validity and chattering
Event-driven switching	Switch controllers by phase and state	Continuity and stability at switching

10.2.4 Virtual Tubes and Behavior-Based Methods: Blending Example

One Euler Step With Opposing Behaviors

$$u_g = 2 - x, \quad u_s = -1, \quad w_g = 0.6, \quad w_s = 0.4$$

$$x = 0: \quad u = 0.6 \times 2 + 0.4 \times (-1) = 0.8 \text{ m/s}$$

$$\Delta t = 0.2 \text{ s}: \quad x^+ = 0.16 \text{ m}$$

u_g: velocity toward the 2 m target. u_s: constant separation velocity -1 m/s.
This counterexample assumes ideal velocity execution where the separation term is constant.

10.2.4 Virtual Tubes and Behavior-Based Methods: Blending Results

Continuous Feedback Stops Short of the Target

$$\dot{x} = 0.8 - 0.6x, \quad x(0) = 0 \Rightarrow x(t) = \frac{4}{3}(1 - e^{-0.6t})$$

Time (s)	Actual Position (m)	Error to 2 m Target (m)
0	0	2
2	0.932	1.068
6	1.297	0.703
10	1.330	0.670

Stopping can leave target error. Combined task and safety behaviors need separate acceptance criteria.

10.2.4 Virtual Tubes and Behavior-Based Methods: Structure Selection

Formation Methods for Different Task Constraints

Method	Main Constraint or Reference	Strength and Main Limit
Leader–follower	Leader state plus relative offset	Direct implementation. Depends on leader and state availability.
Virtual structure	Shared pose plus offsets in the structure frame	Clear geometry. Depends on synchronization and tracking accuracy.
Virtual tube	Feasible region plus local safety relations	Allows deformation. Narrow sections require congestion handling.
Behavior-based methods	Module combination and phase switching	Flexible organization. Recheck properties after combination.

Exercises and Discussion

Exercises: Consensus Values, Offset Signs, and Tracking

1. Initial states are 1, 3, 8 on the fixed, undirected chain 1-2-3 with ideal first-order feedback. Find the consensus value and the result after removing edge 2–3.

2. Actual relative position: −5 m. Desired relative offset: −3 m. Edge weight: 1. Find this edge's velocity contribution.

3. Leader velocity: 1.2 m/s. Gain: 0.6 s^{-1}. No feedforward. Find the steady error and a correction.

Key Answers

Answers: 4 m, +2 m/s, and −2 m

1. Connected graph: initial mean $(1 + 3 + 8)/3 = 4$. Disconnecting node 3 gives 2, 2, 8.

2. $-[-5 - (-3)] = +2$ m/s. At the desired relative position, this contribution must be zero.

3. Steady error: −2 m. Add reference-velocity feedforward with the same sign, retaining negative proportional error feedback.

$$u_i = 1.2 - 0.6e_i \quad \Rightarrow \quad \dot{e}_i = -0.6e_i$$

Section Summary

Models and Conditions Behind Formation Results

Consensus: specify the graph, weights, model, and update conditions before assessing convergence and its value.

Formation: seek consensus after subtracting offsets. Moving references need velocity or higher-order feedforward.

Structure: physical leaders, virtual rigid bodies, feasible tubes, and behavior combinations organize different references and constraints.

Common errors: final spacing does not ensure safety throughout motion. Weighted sums do not retain every property of each behavior.

Check control laws, reference generation, and safety in one closed loop.