Chapter Outline

Vehicle Forces, Coordinates, and Motion Control

Chapter 1 Introduction

1.1 Concepts and Applications

1.2 History and Trends

1.3 Components and Architecture

1.4 Coordinate Systems and Control Principles

1.5 Assembly and Debugging

1.6 RflySim Toolchain

1.7 Learning Path and Experiments

1.8 Exercises and Discussion

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1.4.2 Airframe Configurations and Coordinate Systems
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1.4.1 Comparing Control Principles: Vehicle Overview

Common Elements: Actuators, Media, and Force Application Points

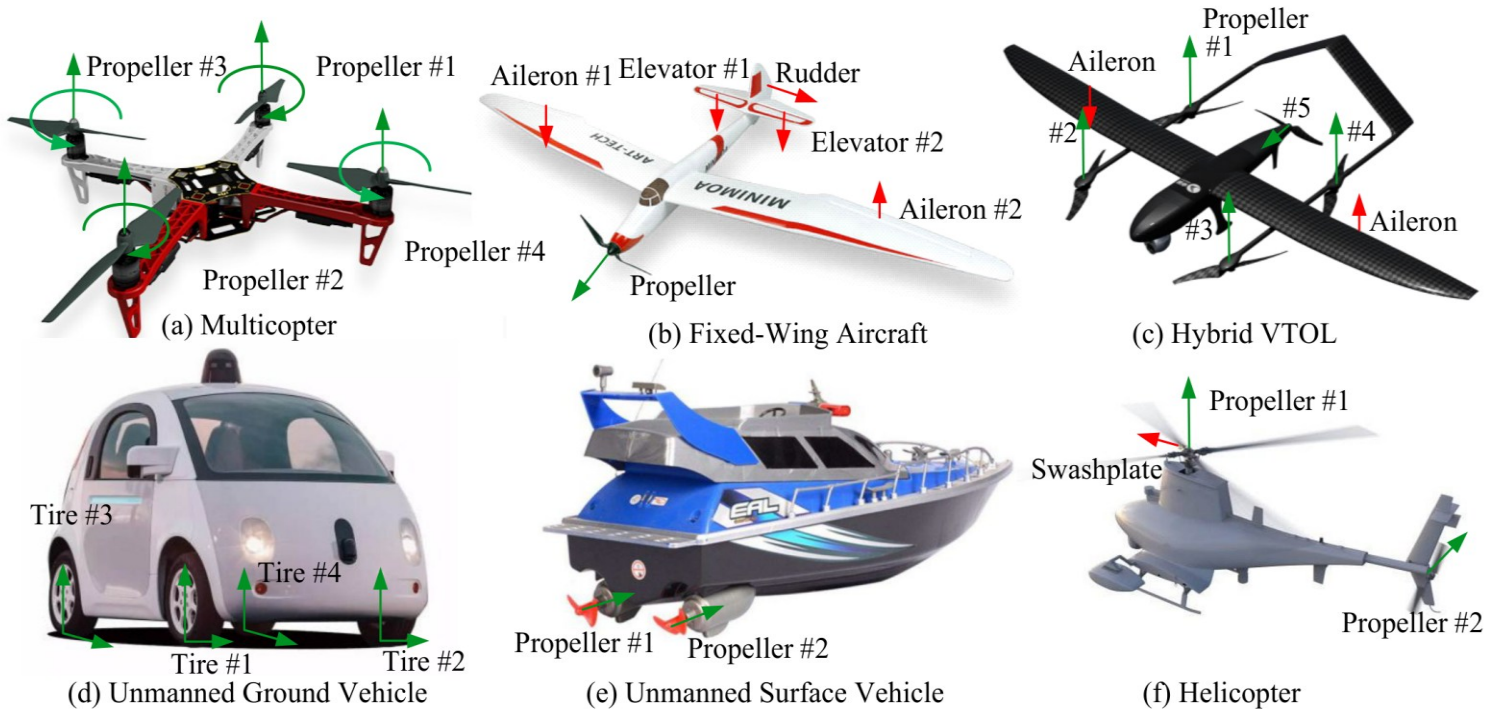


Fig. 1.7 Control Principles of Different Unmanned Vehicles

Actuators generate forces through the surrounding medium. Force location relative to the center of mass affects its moment; different actuators and media need different motion and control methods.

- **Air vehicles:** rotors provide thrust, wings provide lift, and control surfaces adjust moments. VTOL = vertical takeoff and landing.
- **Ground and surface vehicles:** ground forces act on wheels; water forces act on propellers and rudders.

1.4 Coordinate Systems and Control Principles

1.4.1 Comparing Control Principles: Multicopters

Collective Thrust Controls Vertical Motion; Differential Thrust Changes Attitude

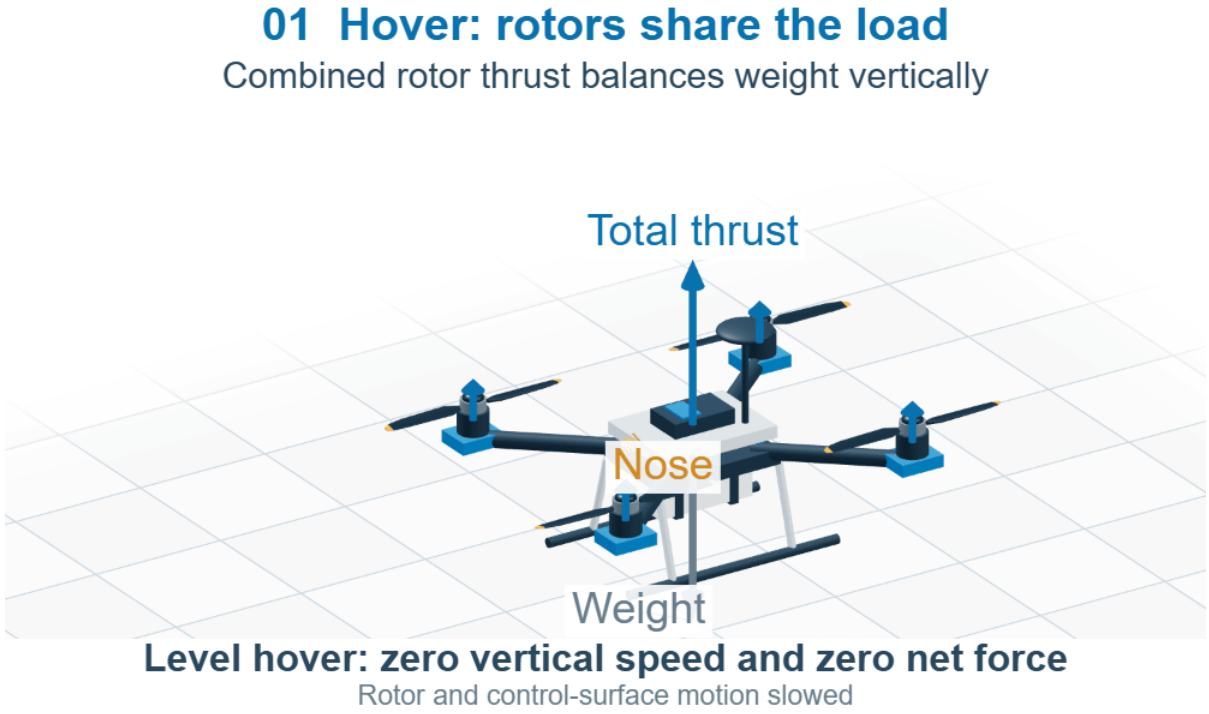


Figure: Multicopter

- **Vertical motion:** near level, vary all rotor thrusts together to change net vertical force. Altitude hold needs zero vertical speed and balanced forces.
- **Translation and braking:** differential thrust first changes pitch or roll. Tilted thrust accelerates; opposite tilt brakes. At steady speed, horizontal thrust equals drag.
- **Yaw:** speed up one counter-rotating group and slow the other. Reaction torque differences change heading at nearly constant total thrust.

A multicopter changes attitude through differential moments, then tilts its thrust to move horizontally.

1.4.1 Comparing Control Principles: Fixed-Wing Aircraft

How Propulsion, Lift, and Control Surfaces Change the Flight Path

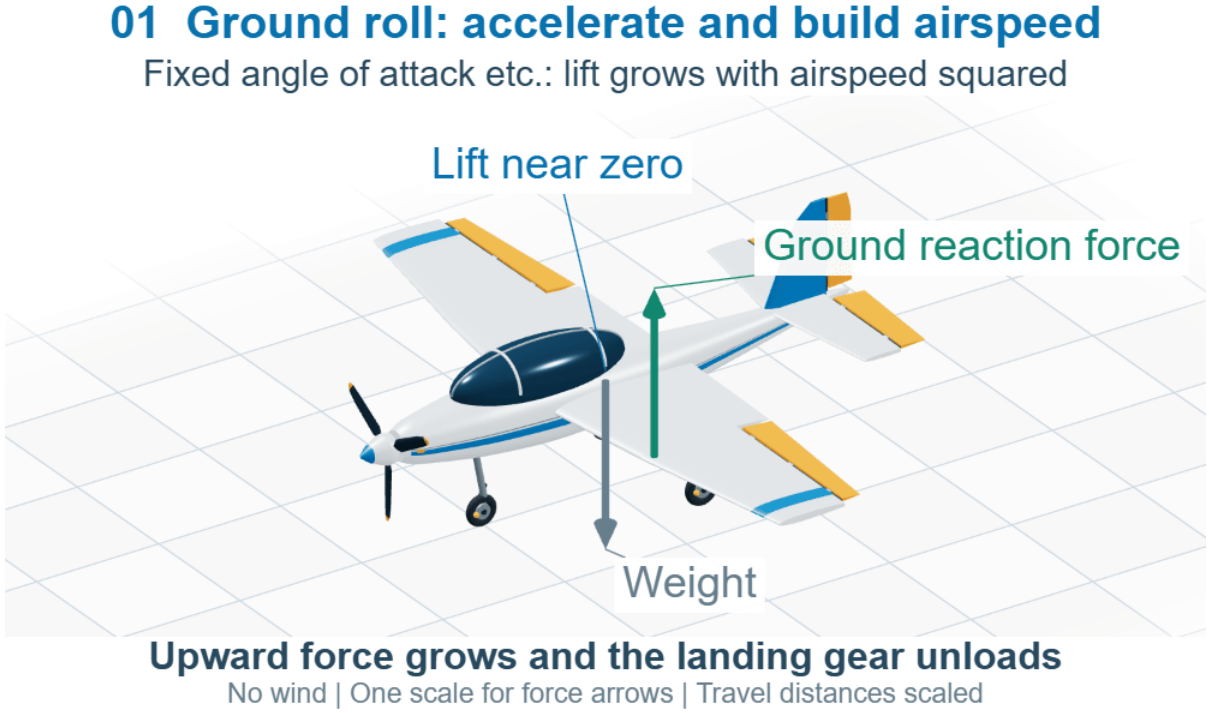


Figure: Fixed-Wing Aircraft

- **Takeoff:** thrust builds airspeed. With angle of attack and other conditions fixed, lift rises with airspeed. Rotate and lift off once takeoff conditions are met.
- **Level flight and stall:** steady level flight balances lift with weight and thrust with drag. Exceeding the critical angle of attack causes stall.
- **Turning:** control surfaces change attitude. Banked lift's horizontal component turns the path; the rudder coordinates yaw.

Relative airflow provides lift and control authority; propulsion and attitude control jointly shape the flight path.

1.4.1 Comparing Control Principles: Fixed-Wing Control Surfaces

Primary Control Axes and Aerodynamic Coupling

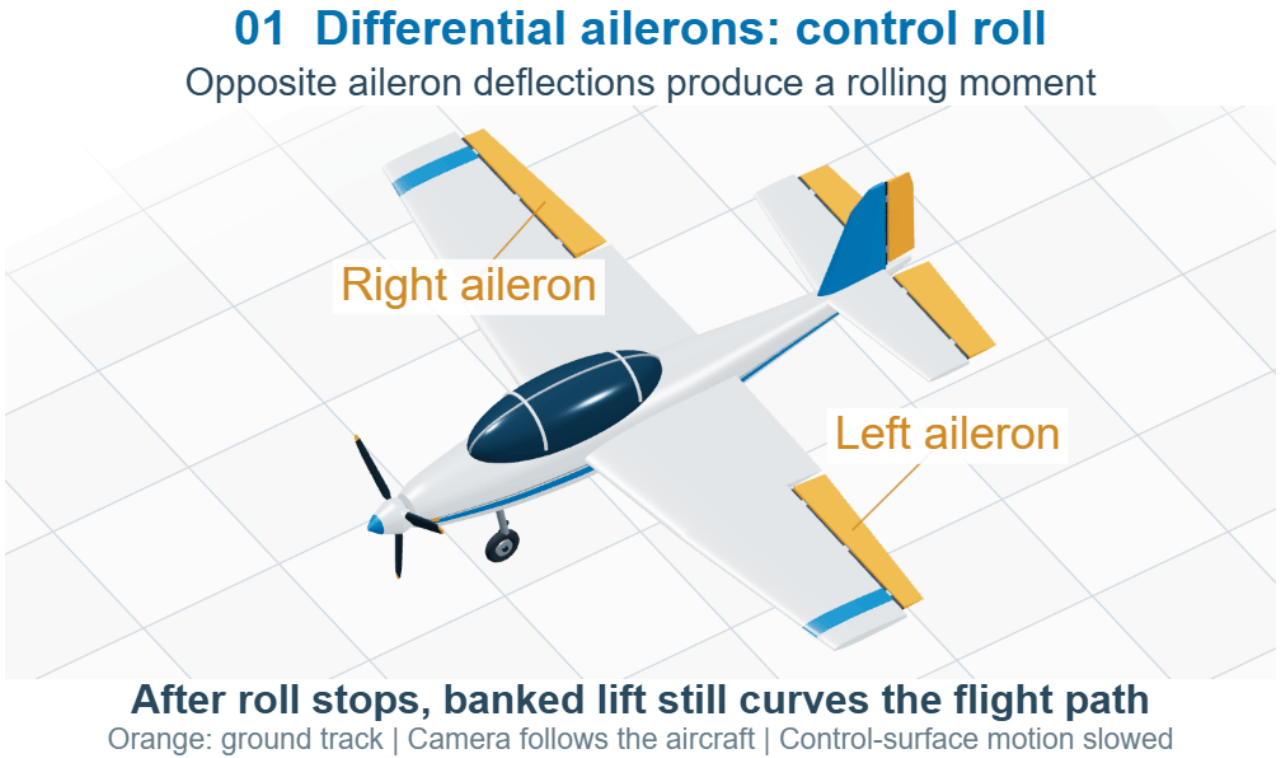


Figure: Three Control Surface Groups; Body Axes: X Forward, Y Right, Z Down

Ailerons: differential trailing-edge deflection mainly rolls the aircraft about body X. Unequal wing drag may also cause yaw.

Elevator: the horizontal tail or equivalent surface changes aerodynamic force, mainly producing pitch moment about body Y.

Rudder: the vertical tail or equivalent surface generates side force, mainly producing yaw about body Z and coordinating turns with ailerons.

1.4.1 Comparing Control Principles: Hybrid VTOL Aircraft

Transition Between Vertical Takeoff and Wing-Borne Cruise

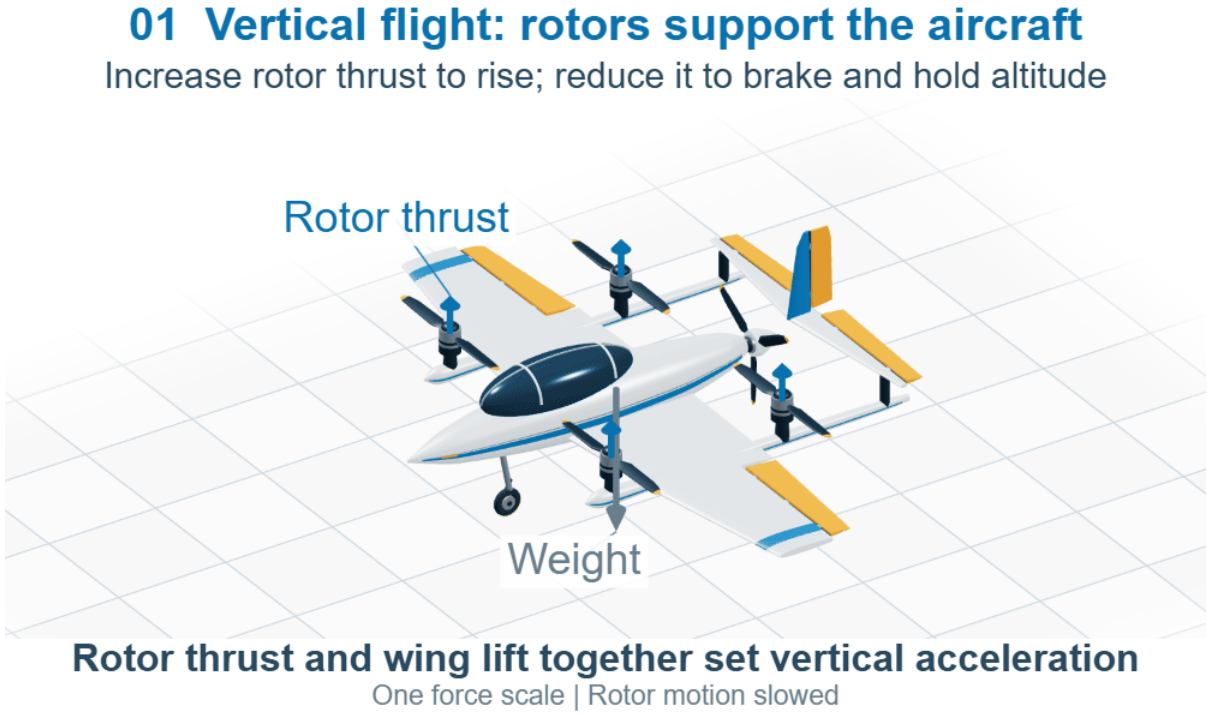


Figure: Hybrid VTOL Aircraft

- **Takeoff and landing:** fixed-wing aircraft often need a runway or aids. Hybrids use lift rotors for vertical takeoff and landing (VTOL); a cruise propulsor accelerates forward.
- **Transition:** wing lift grows with airspeed. Coordinate rotors and control surfaces to hold attitude and altitude.
- **Cruise:** wings carry most weight; thrust overcomes drag. Separate rotors or tilt mechanisms add mass and complicate control.

Hybrid VTOL combines takeoff and cruise advantages at the cost of added mass and transition control.

1.4.1 Comparing Control Principles: Unmanned Ground Vehicles

How Drive and Steering Arrangements Change Planar Motion

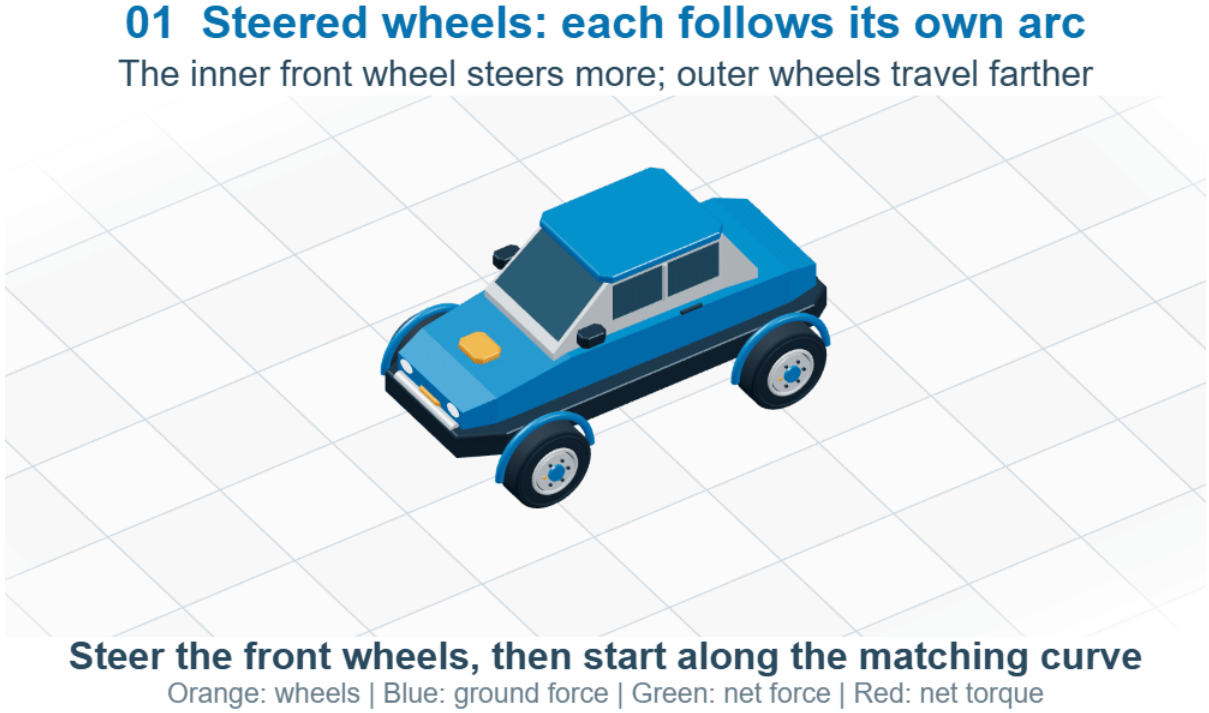


Figure: Unmanned Ground Vehicle

- **Conventional steering:** wheel angles set a curved path. Without lateral slip, turning radius is limited and sideways translation is impossible.
- **Tracked differential steering:** unequal track speeds turn the vehicle. Opposite directions allow pivot turns, with ground slip in practice.
- **Mecanum wheels:** angled rollers and combined wheel speeds allow ideal lateral motion; tires, surface, and load limit real performance.

Steering limits instantaneous motion; wheel-speed allocation must also satisfy contact and traction conditions.

1.4.1 Comparing Control Principles: Unmanned Surface Vehicles

Steering with Differential Thrust and Rudders

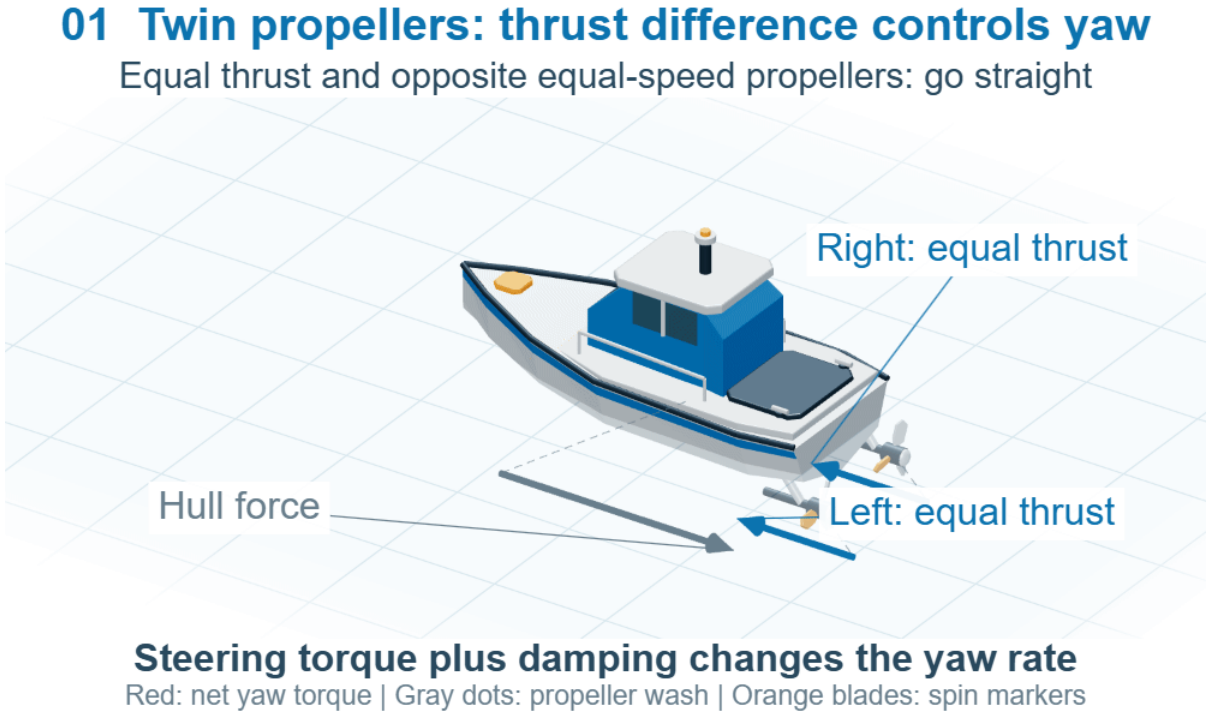


Figure: Unmanned Surface Vehicle

- **Twin propellers:** thrust acts through moment arms. Unequal thrust turns the bow; equal speeds do not guarantee straight motion.
- **Single propeller and rudder:** the propeller provides thrust; water over the rudder produces side force. Authority varies with local flow and operating state.
- **Path feedback:** wind, currents, and hull hydrodynamics affect the path. Heading and position feedback provide continual correction.

A surface vehicle generates the required yaw moment by changing propulsion or rudder side force.

1.4.1 Comparing Control Principles: Unmanned Helicopters

Collective Pitch, Cyclic Pitch, and Tail Rotor Control

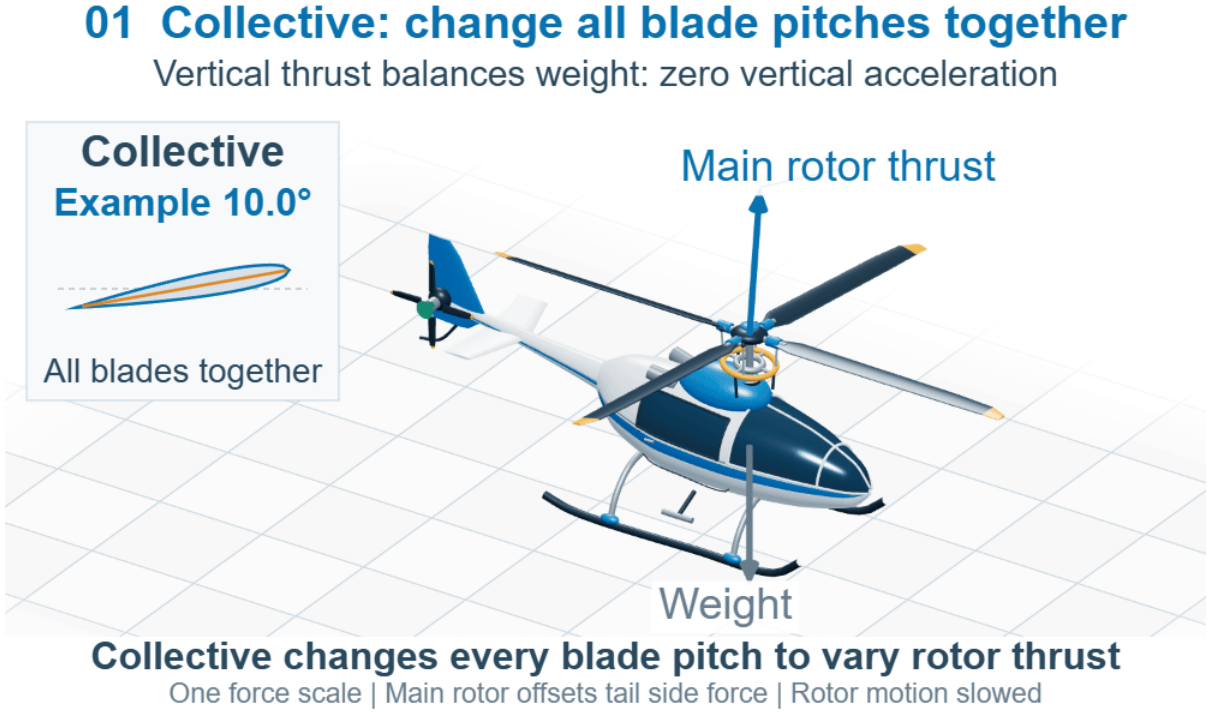


Figure: Helicopter

- **Collective pitch:** change all blade pitches together to adjust main-rotor thrust, net vertical force, and vertical motion.
- **Cyclic pitch:** blade pitch varies periodically with rotor azimuth, tilting the disk and thrust to create horizontal force.
- **Tail rotor:** side thrust and its moment arm balance reaction torque. Increasing collective needs coordinated tail-rotor control; changing the compensation moment controls yaw.

Increasing collective pitch also changes reaction torque, so vertical motion and yaw require coordinated control.

1.4.1 Comparing Control Principles: Other Unmanned Vehicles

Support and Yaw Control by Vehicle Type

> Configurations differ in support and yaw-moment sources.



Coaxial rotors: opposite spin, balanced torque

Vehicle	Source of Lift or Buoyancy	Yaw Control
Single-Rotor Helicopter	Main rotor provides most lift	Tail-rotor thrust adjusts yaw moment
Coaxial Rotors	Both upper and lower rotors provide lift	Change the difference in rotor reaction torque
Unmanned Airship	Mainly air buoyancy	Propellers and control surfaces steer
Underwater Vehicle	Buoyancy plus hydrodynamic forces	Allocate inputs by propeller and control-surface layout

Support determines the basic force balance; yaw control uses adjustable propulsion, rudder forces, or reaction torques.

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1.4.2 Airframes and Coordinate Systems: Symmetry and Center of Mass

Symmetric Layouts, Center of Mass, and Control References

- **Layout:** actuator positions and moment arms set how outputs combine into forces and moments.
- **Symmetry:** equal left–right propulsion and moment arms give symmetric moments and simplify trim.
- **Center of mass:** payload changes shift it; even a symmetric airframe may need unequal trim outputs.
- **Sensor mounting:** inertial sensors near the center of mass experience less rotation-induced acceleration; vibration must still be suppressed.

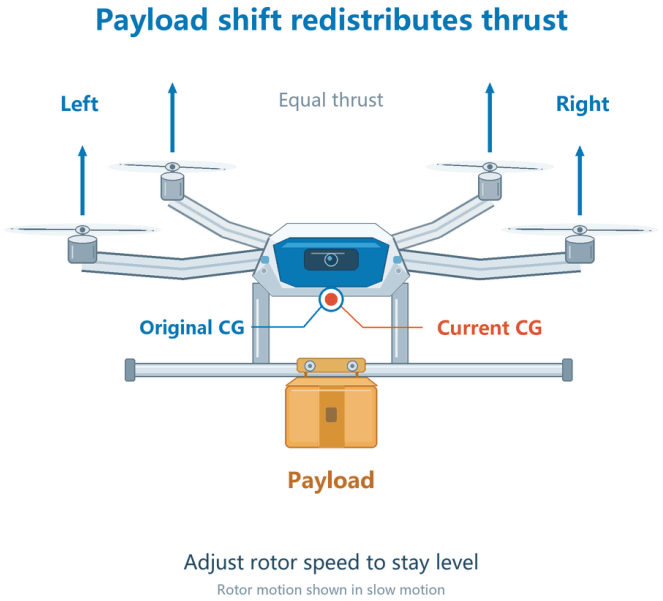


Figure: Payload shifts mass center and moment arms; retrim thrust.

Payload changes shift the center of mass and moment arms; trim outputs must match the actual layout.

1.4.2 Airframes and Coordinate Systems: Nose Direction and Layout

Nose Direction and Airframe Layout: Origin, Axes, and Control References

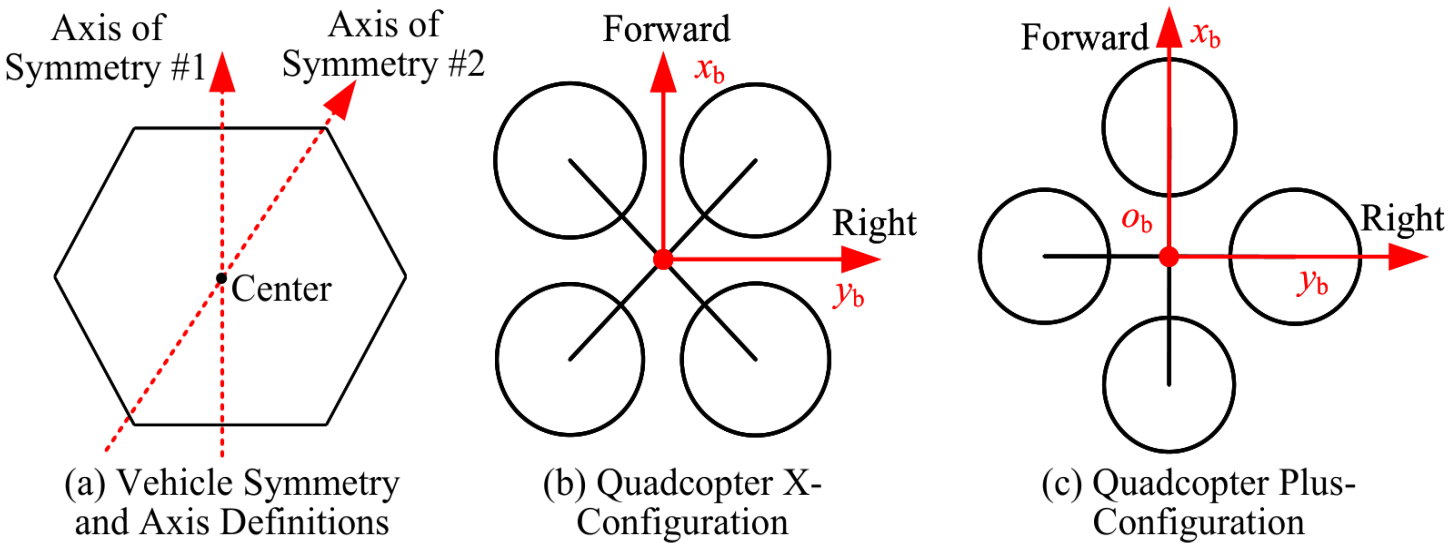


Fig. 1.8 Nose Definition and Multicopter Airframe Layouts

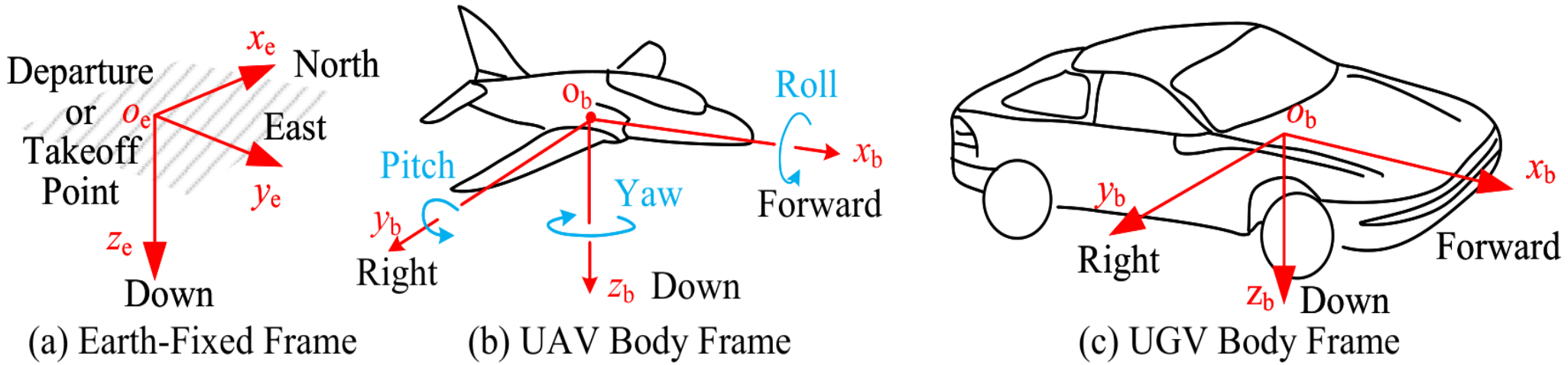
Left–right symmetry helps balance propulsion and payload for trim. Structure, mounting, and configuration define the nose.

- **Nose convention:** X points between arms; plus along an arm.
- **Common reference:** mass-center origin; X forward, Y right, Z down.
- **Control link:** update moment arms and control allocation for new layouts.

The nose defines the forward axis; the origin and remaining axis directions complete the body frame.

1.4.2 Coordinate Systems: Ground and Body Frames

Reference Frame Origins, Axes, and Relative Attitude



(a) Earth-Fixed Frame (b) UAV Body Frame (c) UGV Body Frame

Fig. 1.9 Ground and Body Coordinate Frames

- **Ground reference:** north, east, down; origin at takeoff or another known point.
- **Body reference:** forward, right, down. UAV: unmanned aerial vehicle; UGV: unmanned ground vehicle. Position transforms include translation; direction transforms only rotate.
- **Approximate inertial frame:** at low speed and altitude over a small area, neglect Earth's rotation and use the ground frame as inertial.

1.4.2 Coordinate Systems: Axis Conventions

Two Common Axis Conventions for Ground and Body Frames

> NED and ENU describe ground references; FRD and FLU describe body references.

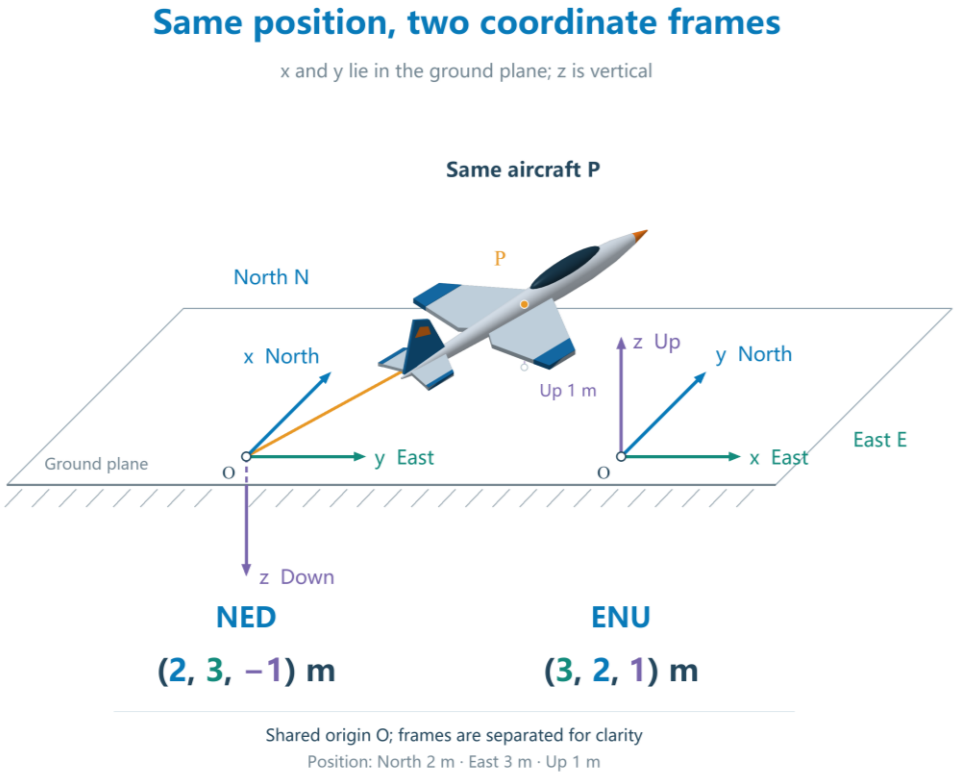
Common Pairing	Ground Axes	Body Axes	Typical Context
North–East–Down (NED) Forward–Right–Down (FRD)	North, East, Down	Forward, Right, Down	Aircraft navigation and flight control For example, PX4 interfaces
East–North–Up (ENU) Forward–Left–Up (FLU)	East, North, Up	Forward, Left, Up	Robot Operating System (ROS) Common conventions

NED to ENU swaps north/east and reverses vertical; FRD to FLU reverses right/down.

1.4.2 Coordinate Systems: Coordinate Transformations

Coordinate Transformations on One Platform: Frames and Data Meaning

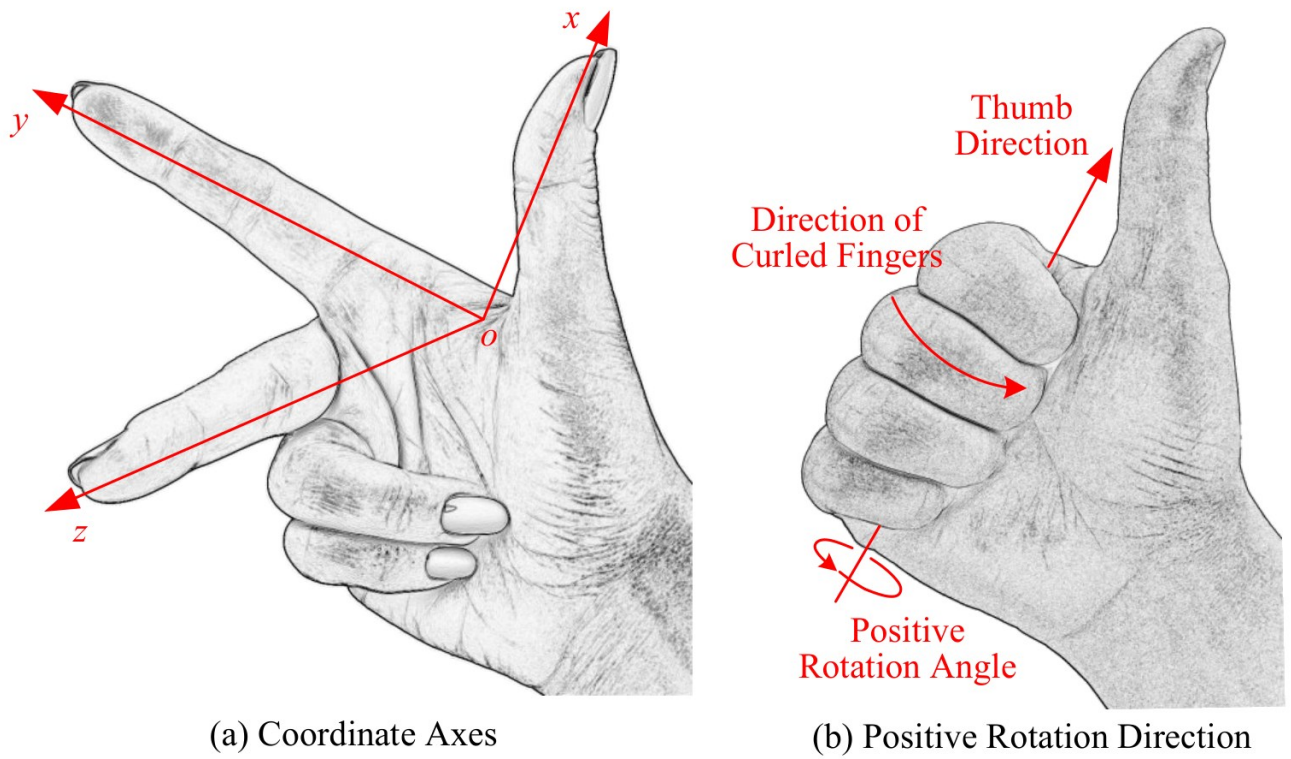
- Interface differences: autopilots and higher-level software may use different frames. Message conventions specify frames and units.
- Ground axes: NED to ENU swaps north/east components and reverses vertical. In NED, z is negative above the origin.
- Body axes: FRD to FLU retains forward and reverses the right and down components.
- Example: an aircraft is 2 m north, 3 m east, and 1 m up. NED: (2, 3, −1) m; ENU: (3, 2, 1) m.



Coordinate transformations preserve physical meaning; axis order, positive directions, and origins define the mapping.

1.4.2 Airframes and Coordinate Systems: Right-Hand Rule

Use the Right Hand to Define Axes and Positive Rotation



(a) Coordinate Axes (b) Positive Rotation Direction

Fig. 1.10 Axes and Positive Rotation Under the Right-Hand Rule

For consistent transformations and calculations, NED, FRD, ENU, FLU, and their derivatives must follow the right-hand rule.

- **Axes:** the right thumb, index, and middle fingers indicate positive X, Y, and Z.
- **Positive rotation:** point the right thumb along the positive axis; curled fingers show positive rotation.
- **Sign check:** fix the viewing direction and positive axis before assigning the sign.

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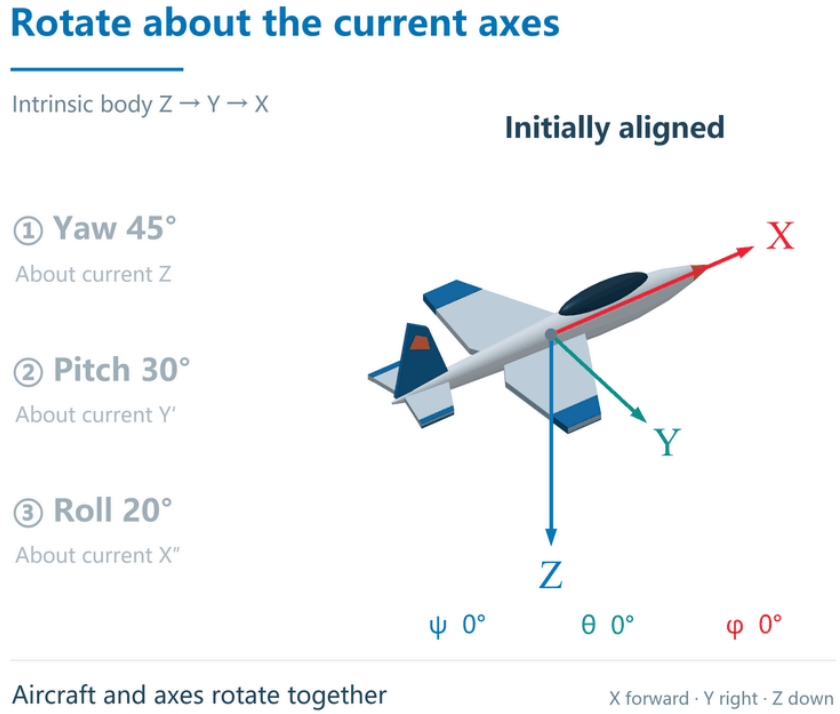
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1.4.3 Attitude Representations: Euler Angle Definitions

Represent Attitude with Three Ordered Rotations

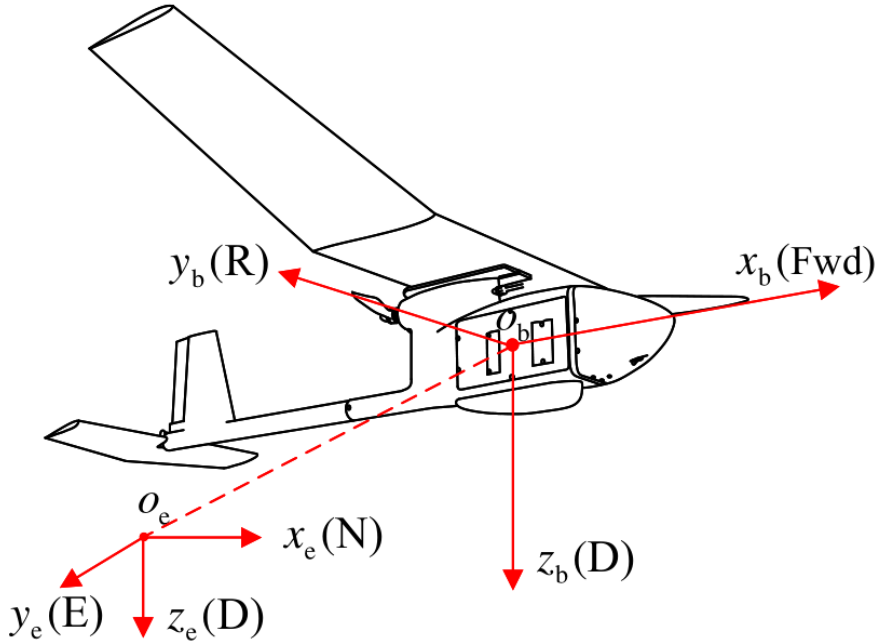
- Rigid-body approximation: treat the vehicle as having a fixed shape. Attitude describes its orientation relative to a reference frame.
- Three rotations: begin with aligned frames. Each rotation uses the updated body axes; this is an intrinsic rotation.
- Convention here: aerospace intrinsic Z-Y-X, with yaw ψ, pitch θ, then roll ϕ.
- Angles and rates: gyroscopes measure body-frame angular velocity, generally different from the individual rates of ψ, θ, and ϕ.



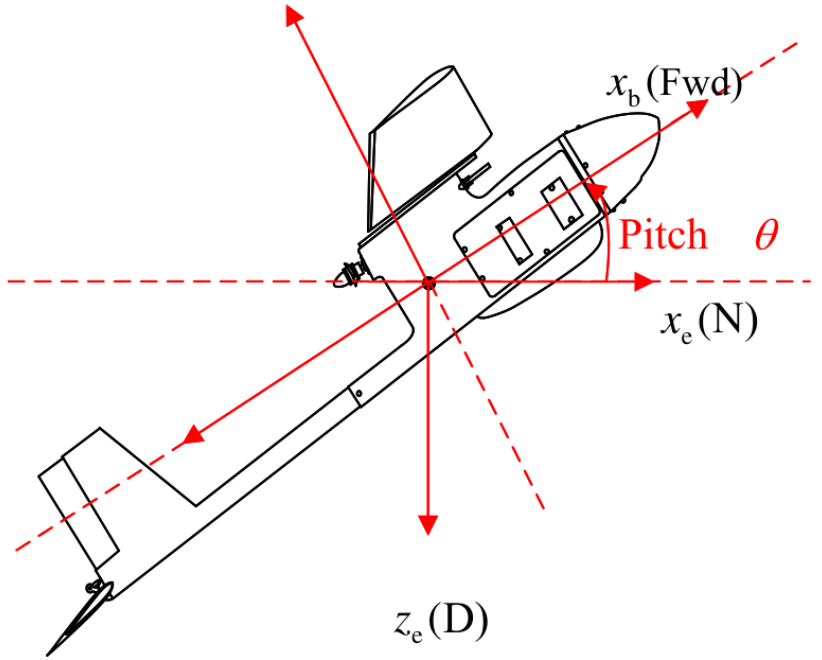
Euler angles represent attitude by ordered rotations; their meaning depends on axis order and positive directions.

1.4.3 Attitude Representations: Pitch

Body Frame and Pitch: Forward–Right–Down and Nose-Up Rotation



(a) Body axes: forward, right, down



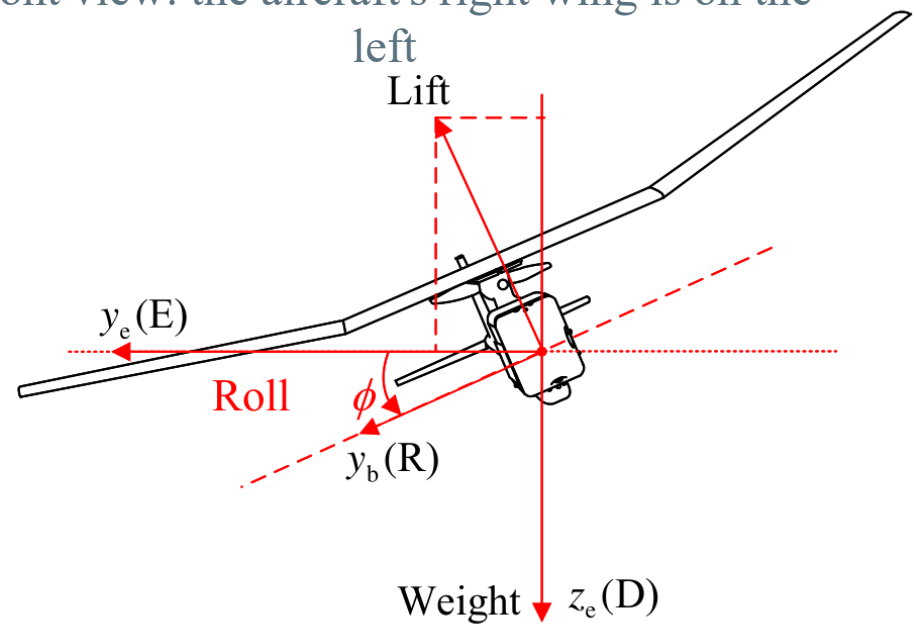
(b) Positive pitch: nose up about body Y

Define the body axes before computing Euler angles; pitch is the nose angle relative to the horizontal plane.

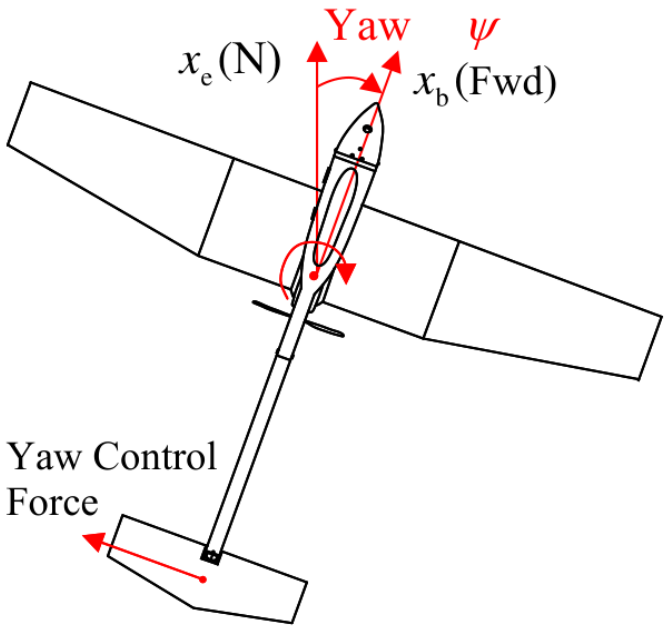
1.4.3 Attitude Representations: Roll and Yaw

Roll and Yaw: Right Roll and Right Yaw Are Positive

Front view: the aircraft's right wing is on the left



(c) Positive roll: roll right about the forward axis



(d) Positive yaw: right turn viewed from above

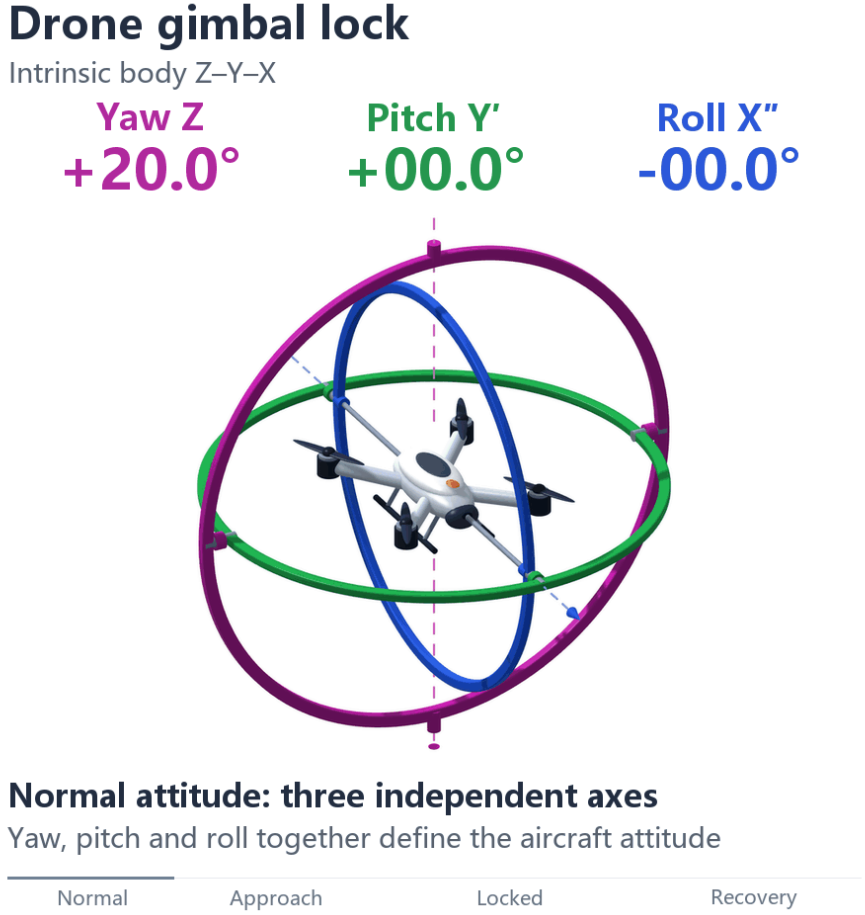
Roll is rotation about the nose axis; yaw is the horizontal heading angle measured from north.

1.4.3 Attitude Representations: Euler Angle Singularity

Euler Angle Singularity at ±90° Pitch

- **Singular condition:** under the Z-Y-X convention used here, the parameters become singular at ±90° pitch.
- **Representation degeneracy:** roll and yaw effects cannot be distinguished independently; multiple angle sets describe the same attitude.
- **Numerical behavior:** near the singular attitude, a small attitude change may cause large Euler angle changes.
- **Gimbal lock:** the inverse angle mapping degenerates; the body remains free to rotate.

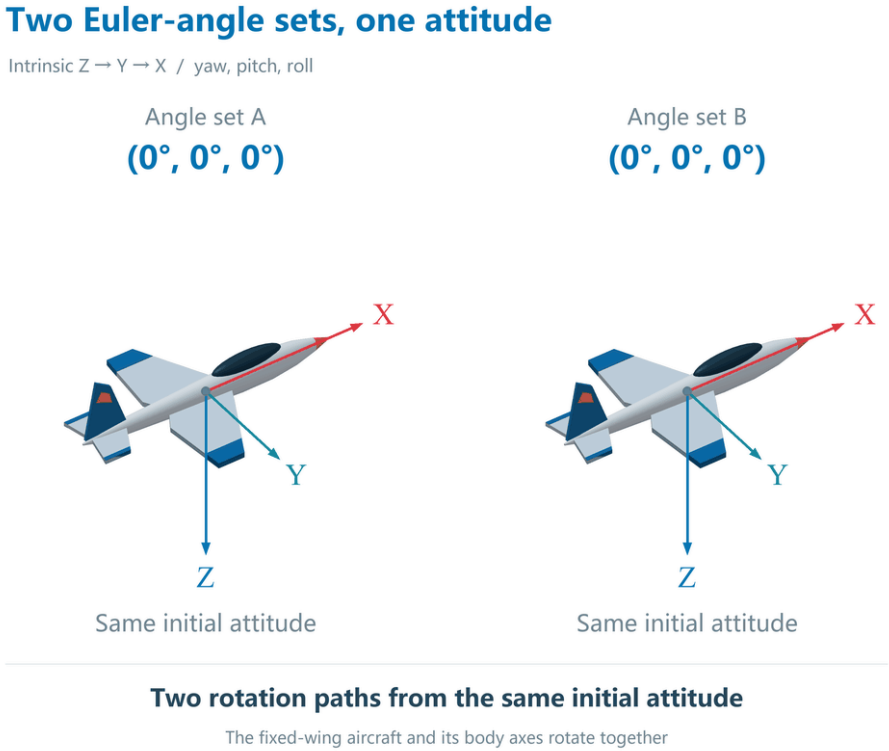
Euler angle singularity couples the parameters while the actual attitude can remain continuous.



1.4.3 Euler Angle Singularity: Inverse Solutions

What an Extra Convention Can Resolve at the Singularity

- Multiple solutions: in (ψ, θ, ϕ) order, (0°, 90°, 0°) and (30°, 90°, 30°) represent the same attitude.
- Chosen convention: at $\theta = 90°$, only $\psi - \phi$ matters. Setting $\phi = 0$ gives one yaw value ψ, but the two angles remain dependent.
- Crossing discontinuity: as pitch rises from 89° to 90°, readings jump from (0°, 89°, 15°) to (−15°, 90°, 0°).
- Continuity: the convention cannot remove the discontinuity at the crossing.



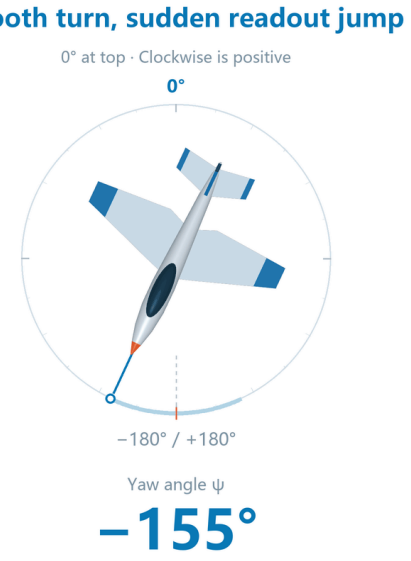
An extra convention selects one angle set; it does not remove the Euler angle singularity.

1.4.3 Euler Angle Singularity: Discontinuous Readings

Euler Angle Discontinuities: Singularities and Range Wrapping

- **Observation:** actual orientation can change smoothly while Euler angles jump when the solution branch changes.
- **Error risk:** direct subtraction or differentiation of jumping angles may produce large errors unrelated to actual motion.
- **Internal representation:** use unit quaternions or rotation matrices for attitude estimation and control to avoid Euler singularities.
- **Display:** convert to Euler angles when needed, specifying the order, ranges, and singularity handling.

Note: yaw wrapping can cause jumps even away from a singularity.



Attitude error describes actual relative rotation; angle wrapping can create false errors under direct subtraction.

1.4.3 Attitude Representations: Other Methods

Three Attitude Representations: Parameters, Constraints, and Uses

> Different parameter counts still describe the same three rotational degrees of freedom.

Method	Representation	Characteristics
Euler Angles	Three ordered rotation angles	Intuitive for display; singular at $\pm 90^\circ$ pitch
Unit Quaternion	Four components with a unit-norm constraint	Nonsingular, smooth interpolation; check signs and component order
Rotation Matrix	3×3 orthogonal matrix with determinant 1	Transforms vectors directly; 9 entries with redundancy; preserve orthogonality

Euler angles are convenient for display; unit quaternions and rotation matrices suit internal rotation calculations.

1.4.3 Attitude Representations: Quaternions

Unit Quaternions: Four Components for Three Rotational Degrees of Freedom

$$\boldsymbol{q} = \left[\cos\frac{\alpha}{2},\ e_x \sin\frac{\alpha}{2},\ e_y \sin\frac{\alpha}{2},\ e_z \sin\frac{\alpha}{2}\right], \quad \|\boldsymbol{e}\| = 1, \quad \|\boldsymbol{q}\| = 1$$

$$\boldsymbol{e} = (0, 0, 1), \quad \alpha = 90^\circ : \quad \boldsymbol{q} \approx [0.707,\ 0,\ 0,\ 0.707]$$

- Axis–angle construction: $\boldsymbol{e}$ is the unit axis and α the angle. Scalar: half-angle cosine; vector: axis times half-angle sine. The 90° Z example turns the nose north to east in NED.
- Unit constraint: squared components sum to 1, leaving three independent rotational degrees of freedom.
- No singularity: each attitude has two representations $\pm\boldsymbol{q}$, not infinitely many. Keep sign continuity during interpolation.
- Order: PX4 (w, x, y, z), scalar first; ROS messages (x, y, z, w). Check the I/O order.

Unit quaternions avoid Euler singularities; component order and rotation convention determine their meaning.

1.4.3 Attitude Representations: Coordinate Transformations

Rotation Matrices: One Vector in Two Coordinate Frames

$$\mathbf{v}_{\mathrm{e}} = R_{\mathrm{b}}^{\mathrm{e}}\mathbf{v}_{\mathrm{b}}$$

b: body frame; e: ground frame

$$R_{\mathrm{b}}^{\mathrm{e}} = R_z(\psi)R_y(\theta)R_x(\phi)$$

Z-Y-X order: yaw, pitch, roll

Multiplication changes a column vector's components, not its geometric meaning.
Columns give body X, Y, and Z directions in the ground frame, respectively.
Apply the rightmost factor first when a composite matrix acts on a column vector.
For an orthogonal rotation matrix, the transpose is the inverse.

Y-only pitch-up of 30°: forward unit vector $[\sqrt{3}/2, 0, -1/2]^{\mathrm{T}}$; negative z means upward.

A rotation matrix changes a vector's coordinate components, not the geometric vector; its transpose gives the inverse.

1.4.3 Attitude Representations: Elementary Rotation Matrices

Single-Axis Rotations: Three Elementary Matrices

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Reading the matrix: in the rotation-axis row and column, the diagonal entry is 1 and the others are 0. The other two components vary with sine and cosine.
- Reading a column: the first column of $R_y(\theta)$, $[\cos\theta\ ,\ 0,\ -\sin\theta]^{\mathrm{T}}$, gives the nose direction after pitching up. Set $\theta = 30°$ for the previous result.
- Single-axis examples: $R_z(90°)\,[1,\ 0,\ 0]^{\mathrm{T}} = [0,\ 1,\ 0]^{\mathrm{T}}$ turns the nose from north to east. $R_x(\phi)\,[1,\ 0,\ 0]^{\mathrm{T}} = [1,\ 0,\ 0]^{\mathrm{T}}$ shows that roll leaves the nose direction unchanged.

A single-axis rotation changes only the other two components; combine rotations in the specified order.

1.4.3 Attitude Representations: Numerical Example

Rotation Order Matters: Full Matrix and a Two-Angle Check

$$R_{\mathrm{b}}^{\mathrm{e}} = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix}$$

$c_\beta = \cos\beta, \quad s_\beta = \sin\beta, \quad \beta \in \{\phi, \theta, \psi\}$

$R_{\mathrm{e}}^{\mathrm{b}} = (R_{\mathrm{b}}^{\mathrm{e}})^{\mathsf{T}}$

Example: in NED, the nose points east and 30° up

$\psi = 90^\circ, \quad \theta = 30^\circ, \quad \phi = 0^\circ$

Wrong order $R_y R_z$ gives $[0, 1, 0]^{\mathsf{T}}$: east, level, no pitch-up.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{\mathrm{b}} \longmapsto \begin{bmatrix} 0 \\ \sqrt{3}/2 \\ -1/2 \end{bmatrix}_{\mathrm{e}}$$

Composite rotations generally cannot be reordered; use a vector to check the resulting attitude.

1.4.3 Attitude Representations: Rotation Order

Rotation Order Changes Nose Direction

Compare the forward and lateral body axes to distinguish pointing changes from body rotation.

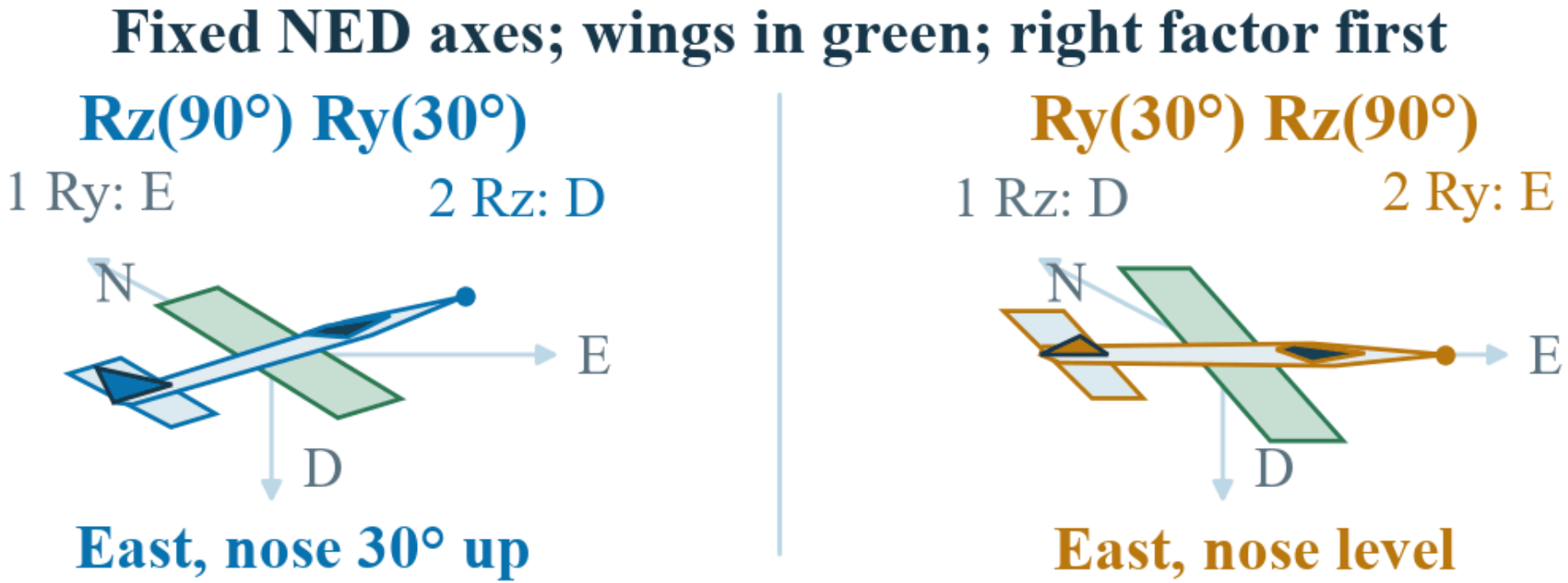


Figure: The same two rotations applied in different orders to the body forward vector (NED schematic)

Chapter Outline

Vehicle Forces, Coordinates, and Motion Control

Chapter 1 Introduction

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1.2 History and Trends

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1.4 Coordinate Systems and Control Principles

1.5 Assembly and Debugging

1.6 RflySim Toolchain

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1.4.1 Differences in Control Principles
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1.4.2 Airframe Configurations and Coordinate Systems
Textbook: Secs. 1.2.3.1–1.2.3.2

1.4.3 Attitude Representations
Textbook: Secs. 1.2.3.3–1.2.3.4

1.4.4 Multicopter Control Principles
Textbook: Sec. 1.2.3.5

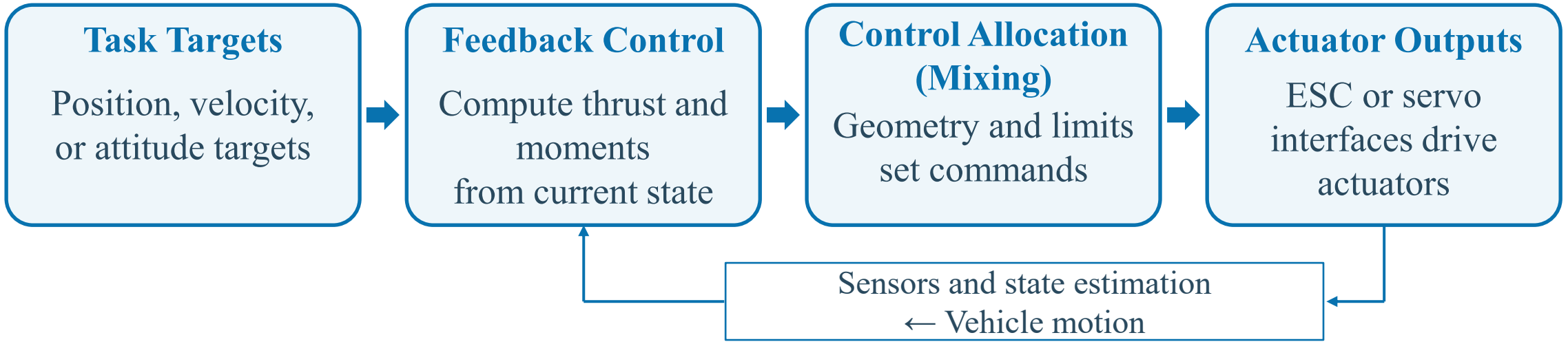
1.4.5 Underactuated Systems
Textbook: Sec. 1.2.3.6

1.4.4 Multicopter Control Principles: Closed-Loop Overview

Feedback and Control Allocation: Targets, Commands, and Motion

> Feedback uses targets and state to compute demands; allocation turns them into individual actuator commands.

Forward arrows: commands; lower loop: motion feedback. ESC: electronic speed controller.

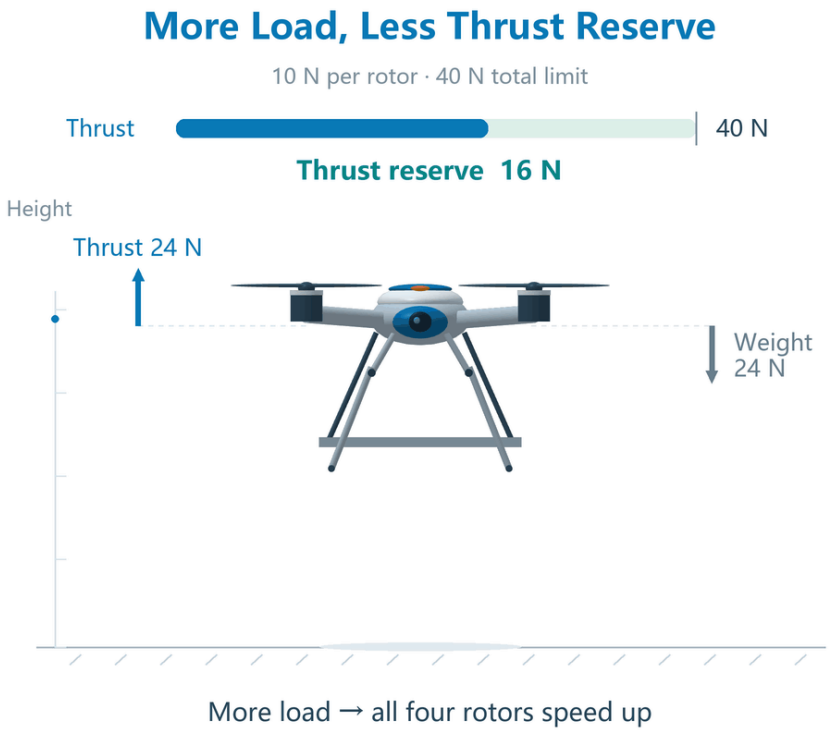


Feedback computes required forces and moments; allocation is constrained by actuator capability.

1.4.4 Multicopter Control Principles: Control Allocation

Allocate Desired Thrust and Moments to Actuators

- **Control objective:** upstream control specifies desired thrust and moments; the allocator determines each actuator's output.
- **Geometry:** actuator positions, moment arms, and directions determine their contributions to resultant force and moment.
- **Control effectiveness:** arrange each actuator's thrust and moment coefficients by actuator number to form the allocation matrix.
- **Limits:** at 10 N per rotor, four rotors provide at most 40 N. A 44 N request requires a trade-off and saturation feedback.

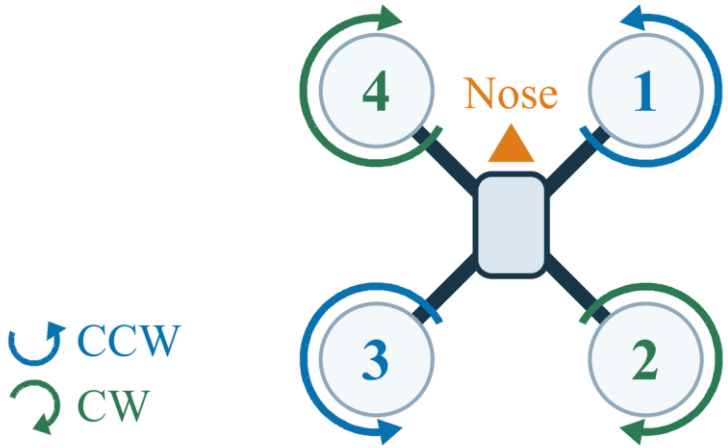


Layout and actuator effectiveness define allocation; output limits bound achievable thrust and moments.

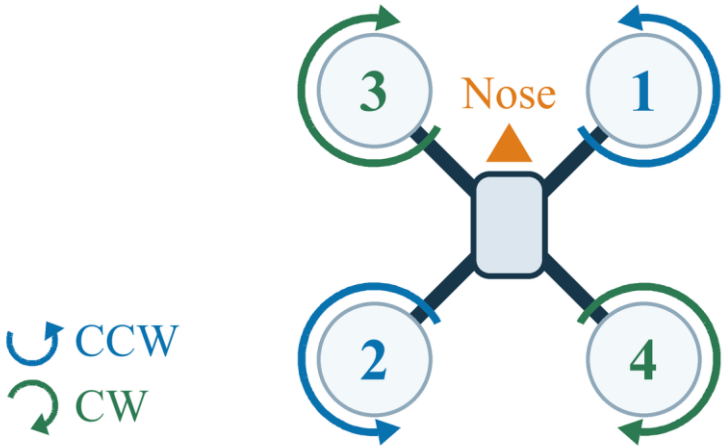
1.4.4 Multicopter Control Principles: Rotor Numbers and Spin Directions

Mapping Textbook Rotor Numbers to the PX4 Example

> X quadcopter viewed from above; the actual mapping follows the selected airframe configuration.



Textbook: 1, 3 counterclockwise (CCW); 2, 4 clockwise (CW)
1 front right, 2 rear right, 3 rear left, 4 front left



PX4 Quad X: 1, 2 CCW; 3, 4 CW
1 front right, 2 rear left, 3 front left, 4 rear right

Rotor positions, numbers, and spin directions define allocation; output-port settings determine the actual control channels.

1.4.4 Multicopter Control Principles: Force Model

Quadcopter Forces: Rotor Thrust, Reaction Torque, and External Forces

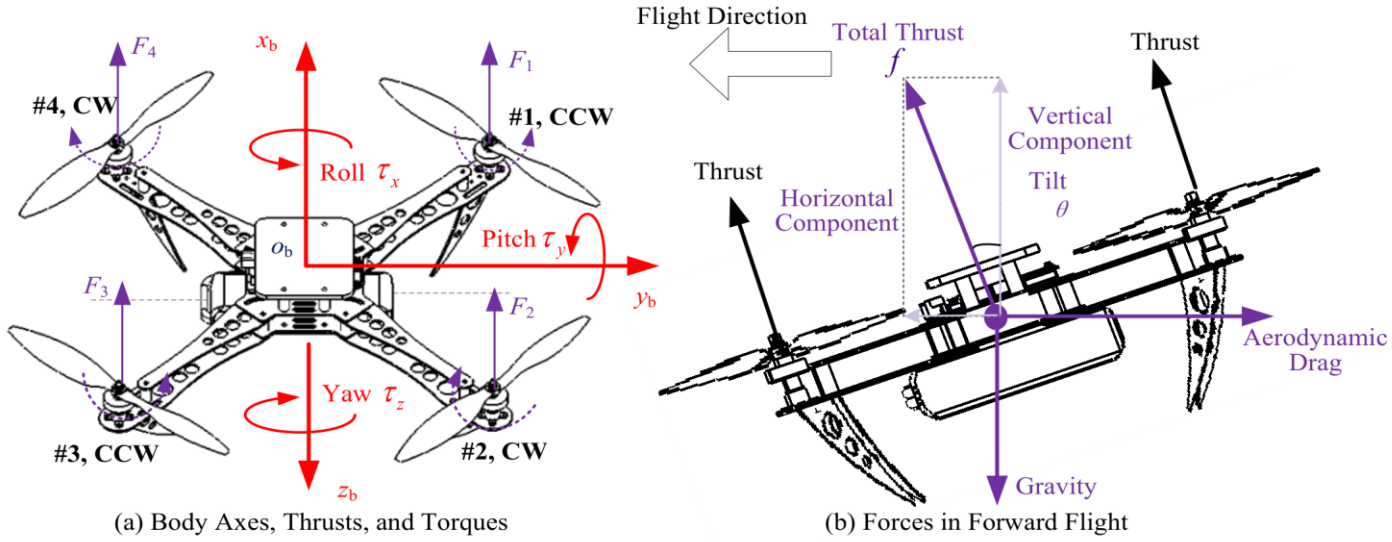


Fig. 1.12 Multicopter Force Model

Axial rotor thrust creates roll and pitch moments through its mounting arm. Rotor spin in air exerts body reaction torque opposite to the spin.

- **Symbols:** red curved arrows show moments; purple dashed lines show rotor spin.
- **Moment sources:** thrust acts through moment arms; reaction torque opposes rotor spin.
- **Forward flight:** resolve thrust in the right figure and compare its components with drag and weight.

1.4.4 Multicopter Control Principles: Combining Thrust and Moments

Four Control Inputs Combine Rotor Thrusts and Reaction Torques

$$f = F_1 + F_2 + F_3 + F_4, \qquad \tau_x = L\sin 45^\circ\ (F_3 + F_4 - F_1 - F_2)$$

$$\tau_y = L\sin 45^\circ\ (F_1 + F_4 - F_2 - F_3), \qquad \tau_z = M_1 + M_2 + M_3 + M_4$$

F_i, M_i: rotor i thrust and reaction torque; L: motor-to-body-center distance; control vector $\boldsymbol{u} = [f, \tau_x, \tau_y, \tau_z]^{\mathrm{T}}$.

- Hover reference: $G = 20$ N, $L = 0.25$ m, 5 N per rotor: $f = 20\ \mathrm{N} = G$. Front/rear and left/right thrusts match; diagonal reaction torques cancel: $\tau_x = \tau_y = \tau_z = 0$.
- One-step calculation (nose-down): rear rotors 2, 3 rise to 5.5 N; front rotors 1, 4 fall to 4.5 N. f stays 20 N, $\tau_x = 0$; $\tau_y = 0.25\sin 45^\circ\ (4.5 + 4.5 - 5.5 - 5.5) \approx -0.35\ \mathrm{N \cdot m}$. Negative means nose-down. Each diagonal group has one increase and one decrease, so $\tau_z \approx 0$.

Collective changes adjust total thrust; differential groups adjust moments. Control allocation solves these relations in reverse.

1.4.4 Multicopter Control Principles: Rotor Groups

Quadcopter Groups: Moments First, Motion Next

> Textbook numbering: 1 front right, 2 rear right, 3 rear left, 4 front left. Adjustments below are relative to a reference state.

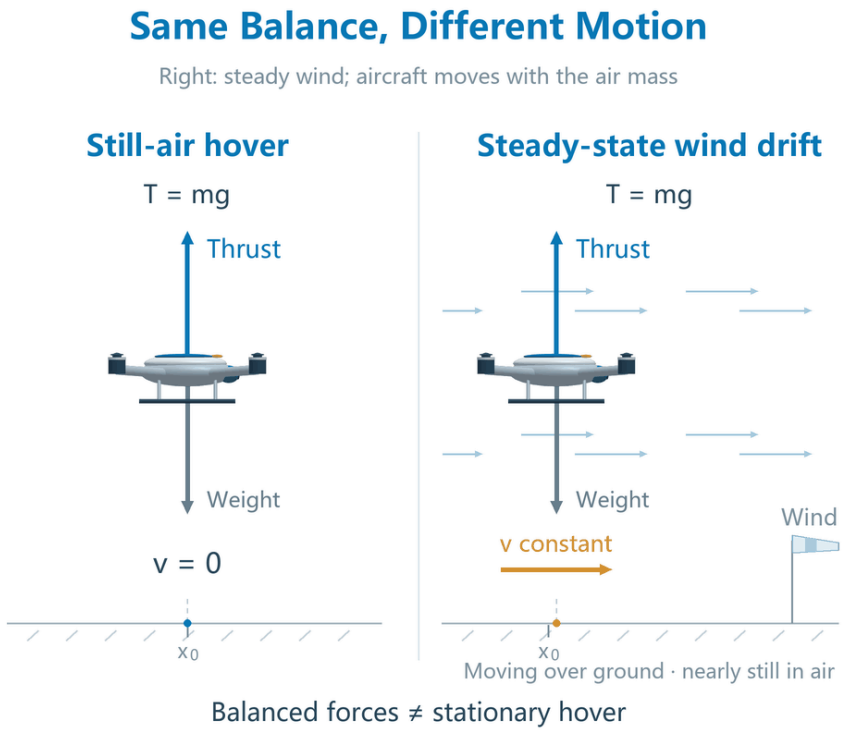
Desired Tendency	Increase Thrust	Decrease Thrust
Upward acceleration	All four rotors	None (relative to reference)
Nose-down	Rear group 2, 3	Front group 1, 4
Right roll	Left group 3, 4	Right group 1, 2
Positive yaw (right)	Counterclockwise group 1, 3	Clockwise group 2, 4

Differential rotor groups first change moments; horizontal motion also requires attitude change and the appropriate force balance.

1.4.4 Multicopter Control Principles: Hover Balance

Hover and Wind Drift: Balanced Forces at Different Velocities

- **State conditions:** hover holds ground-relative position and attitude constant, with zero translational and angular velocity.
- **Force balance:** in both near-level steady states, thrust balances weight. During drift, the vehicle is nearly stationary relative to the air.
- **Moment balance:** net roll, pitch, and yaw moments must all balance; reaction torque cancellation is only one part.
- **Model boundary:** without position feedback, steady wind drift is possible. Zero net force means only constant velocity.

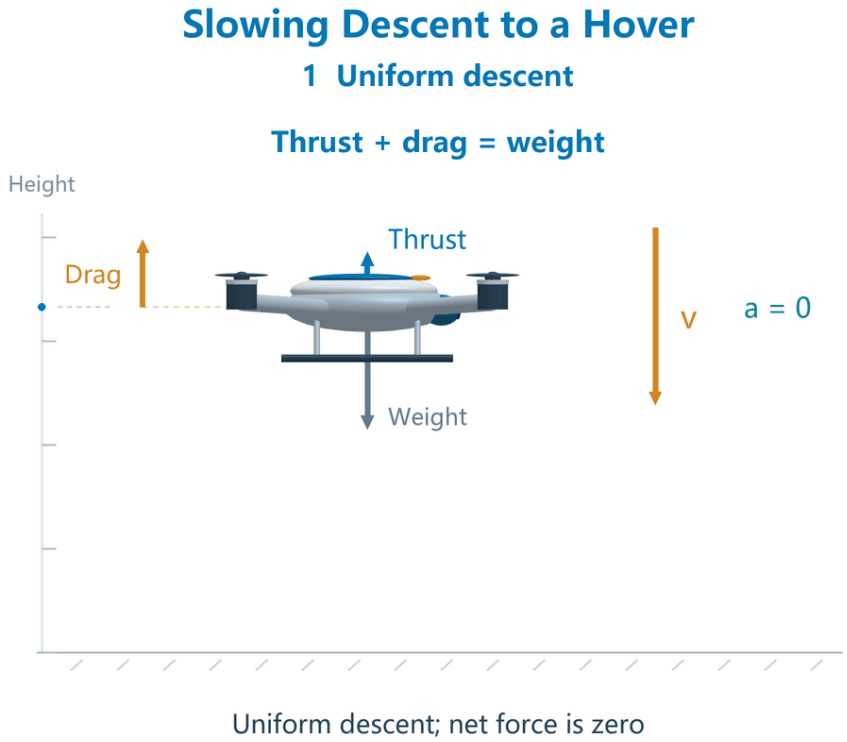


Balanced forces allow constant-speed wind drift; hover also requires zero ground-relative velocity.

1.4.4 Multicopter Control Principles: Vertical Motion and Total Thrust

Vertical Motion: Collective Thrust with Velocity and Altitude Feedback

- **Collective adjustment:** in symmetric, near-level conditions, changing all rotor thrusts together mainly changes net vertical force.
- **Acceleration:** drag points upward during descent. If thrust plus drag exceeds weight, acceleration is upward.
- **Velocity:** balanced forces allow constant-rate descent; upward net force reduces descent speed.
- **Altitude hold:** use altitude and vertical-velocity feedback to adjust output, ultimately reaching zero velocity and balanced forces.



Net vertical force sets acceleration; altitude hold also requires eliminating altitude and vertical-velocity errors.

1.4.4 Multicopter Control Principles: Altitude Feedback Loop

How Altitude Error Becomes Four Rotor Thrusts

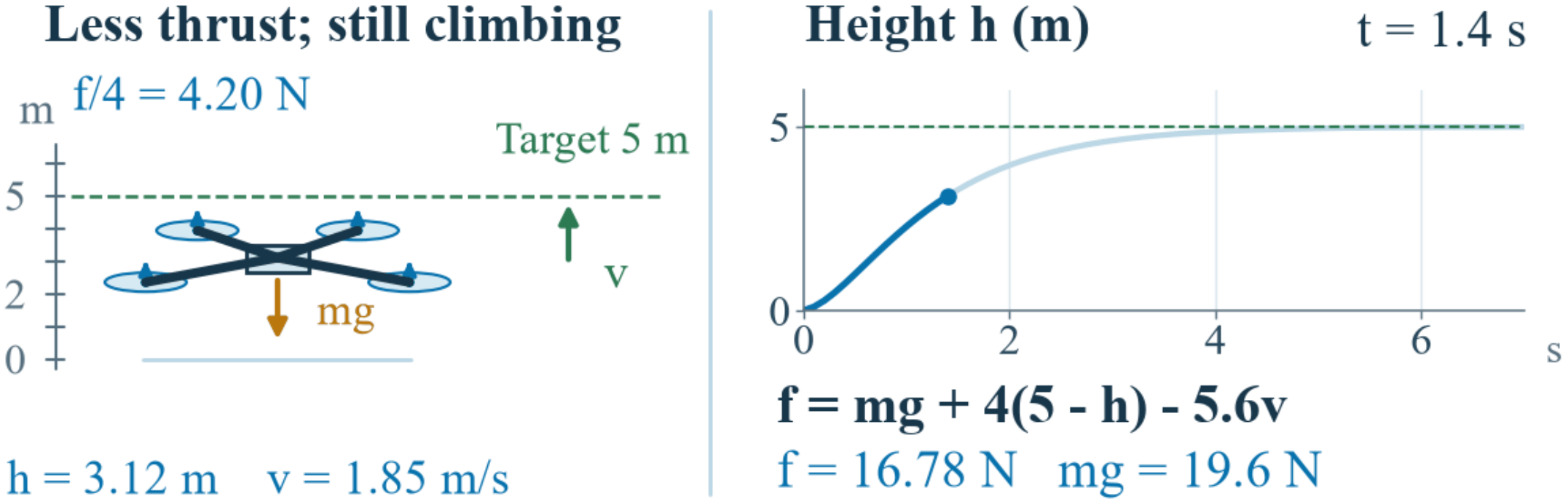


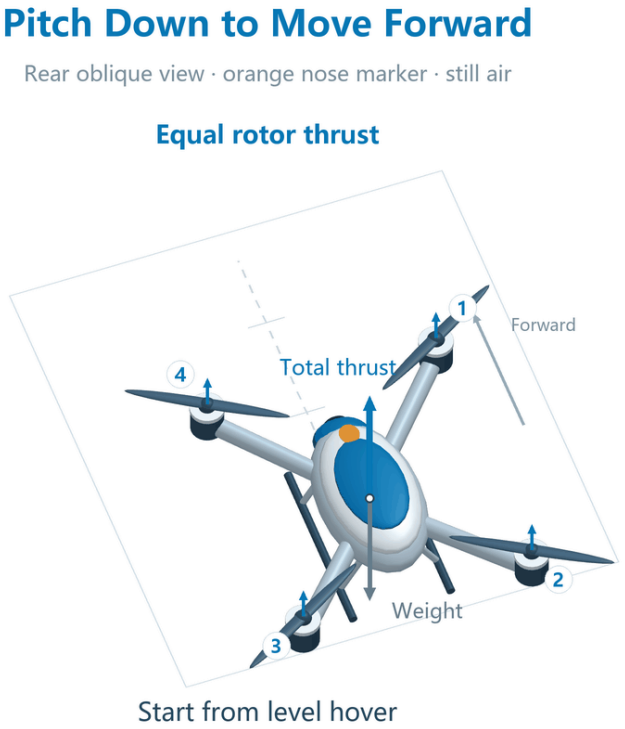
Figure: Sample-by-Sample Altitude Feedback (Teaching Simulation)

The initial error increases total thrust; near the target, thrust returns to the hover value.

1.4.4 Multicopter Control Principles: Longitudinal Motion and Pitch

Pitch Redirects Thrust to Create a Longitudinal Component

- **Initiate pitch:** using textbook numbers, increase rear rotors 2, 3 and decrease front rotors 1, 4 to pitch nose-down.
- **Redirect thrust:** once tilted, total thrust has forward and upward components.
- **Hold attitude:** at the target tilt, reduce pitch rate and balance moments.
- **Hold altitude and speed:** vertical thrust balances weight. Horizontal thrust above drag accelerates; only equality permits constant speed.

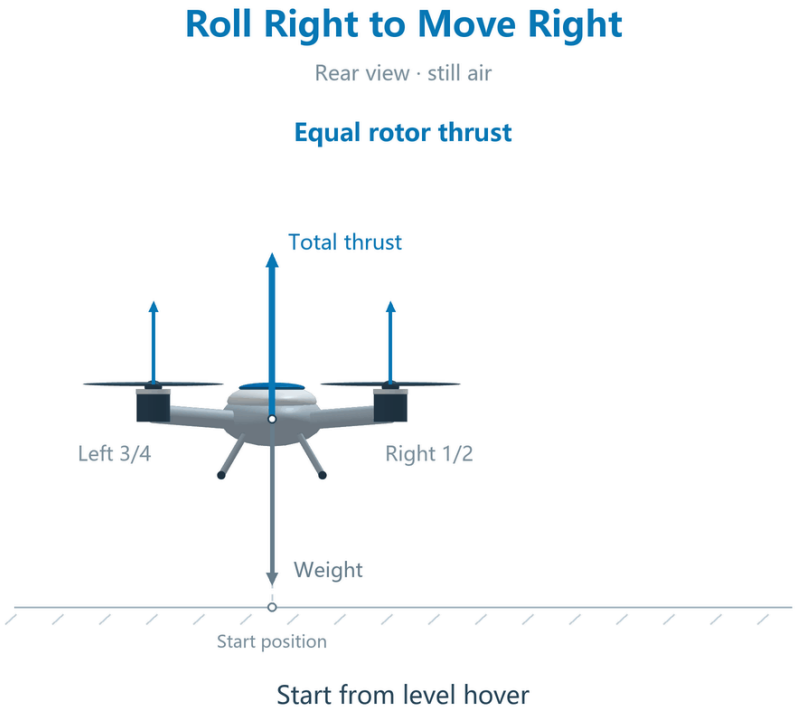


Nose-down tilt gives forward thrust; constant speed still depends on net horizontal force.

1.4.4 Multicopter Control Principles: Lateral Motion and Roll

Roll Redirects Thrust to Create a Lateral Component

- **Initiate roll:** increase left rotors 3, 4 and decrease right rotors 1, 2 to produce right roll.
- **Lateral component:** total thrust tilts with the body, gaining a lateral horizontal component in the ground frame.
- **Hold the target:** at the desired roll angle, the controller must reduce angular velocity and balance roll moment.
- **Motion limits:** compensate vertical thrust to hold altitude. Lateral velocity also depends on drag, wind, and velocity feedback.

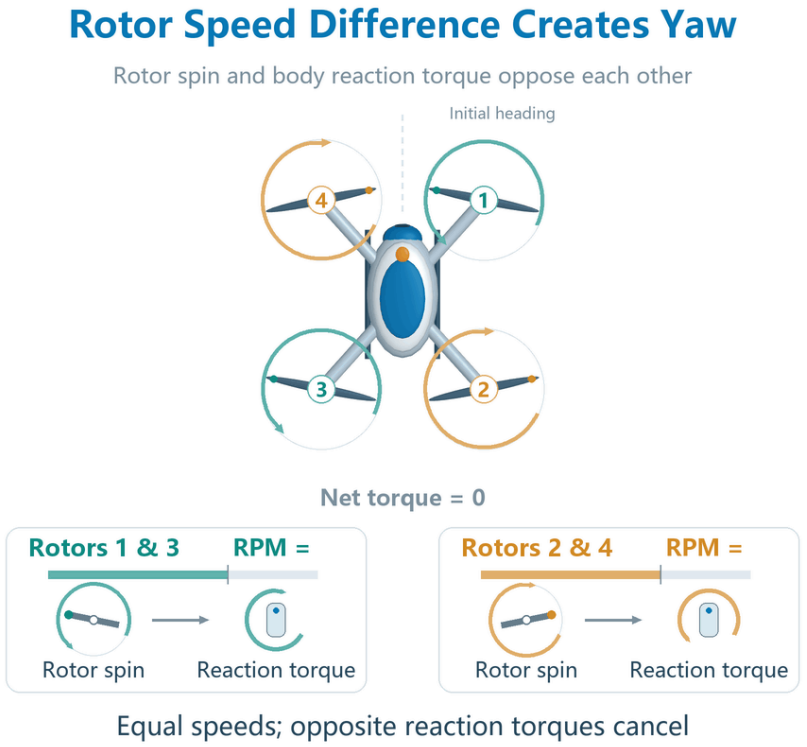


Left–right differential thrust changes roll; tilted thrust then changes lateral velocity.

1.4.4 Multicopter Control Principles: Yaw Moment

Change Heading Through Differential Reaction Torque

- **Moment source:** rotors apply torque to air, which reacts oppositely on the body. Spin direction determines the sign.
- **Right-yaw example:** viewed from above, increase counterclockwise rotors 1, 3 and decrease clockwise rotors 2, 4.
- **Thrust coordination:** thrust is generally nonlinear in rotor speed. Equal speed increases and decreases do not guarantee constant total thrust.
- **Tracking and stopping:** adjust net moment using angular-rate and heading errors to follow the target and stop when required.

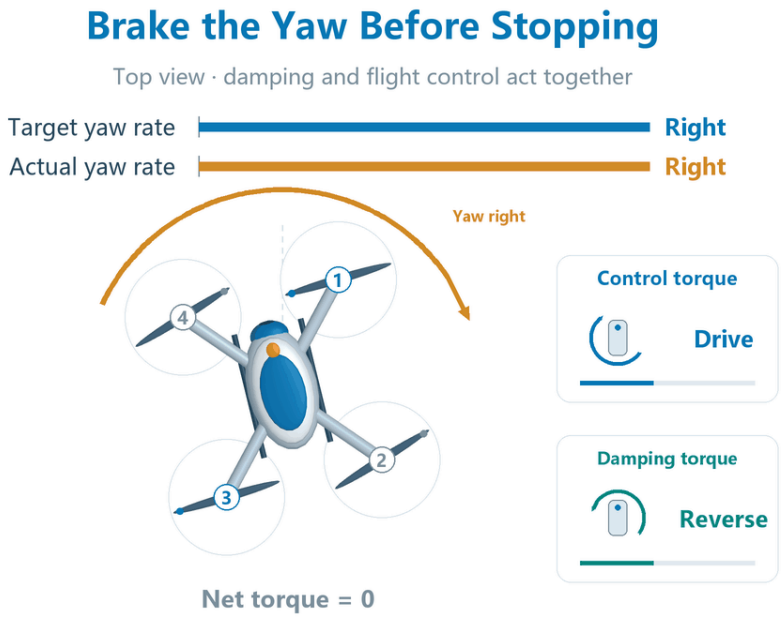


Yaw uses differential reaction torques; maintaining total thrust requires coordination based on rotor-group effectiveness.

1.4.4 Multicopter Control Principles: Yaw-Rate Stabilization

Stabilizing Yaw Rate Through Damping and Feedback

- **Starting rotation:** a nonzero net yaw moment changes yaw rate.
- **Damping:** air and rotors can provide damping, varying with operating conditions and rotor speed.
- **Steady state:** yaw rate can remain constant when driving, damping, and other moments balance.
- **Active control:** flight control adjusts outputs from rate error. Do not rely on passive damping alone to limit yaw rate.



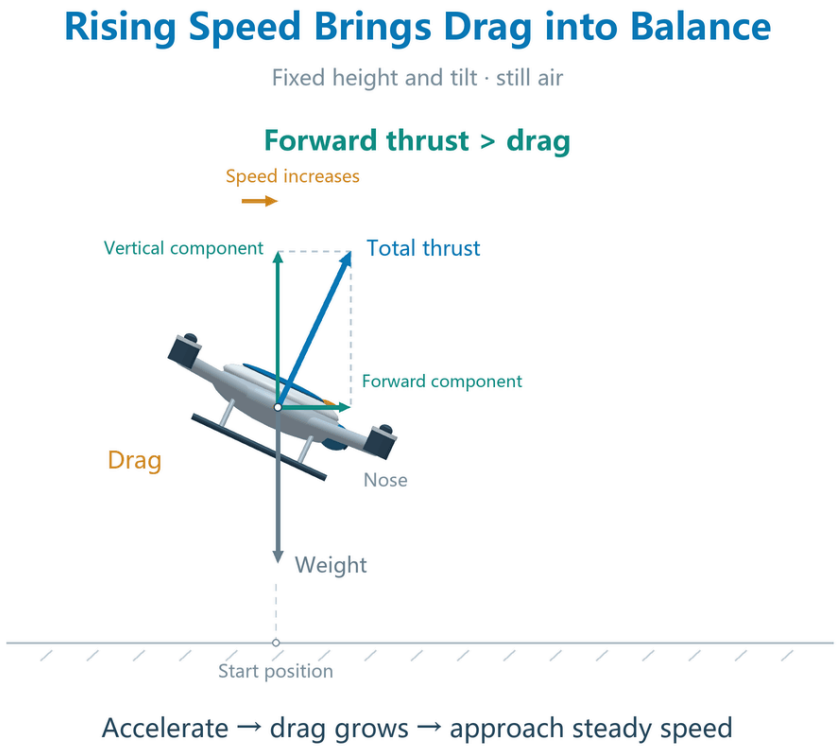
Steady right yaw: drive torque balances damping

Net yaw moment changes angular velocity; damping or feedback must balance moments to maintain a steady rate.

1.4.4 Multicopter Control Principles: Forward-Flight Forces

Tilt and Steady Speed: Vertical and Horizontal Force Balance

- Simplified conditions: no roll or wind; straight forward flight at constant altitude. θ is forward-tilt magnitude; D is aerodynamic drag.
- Vertical balance: $f\cos\theta = G$. Larger θ needs greater total thrust f to hold altitude.
- Horizontal balance: $f\sin\theta = D$ gives constant speed; exceeding D accelerates. Dividing gives $\tan\theta = D/G$.
- Tilt limit: larger θ gives higher steady speed; excessive tilt saturates some motors and removes control margin.



In level forward flight, tilt sets thrust components; horizontal force balance sets steady speed.

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1.4.4 Multicopter Control Principles
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1.4.5 Underactuated Systems
Textbook: Sec. 1.2.3.6

1.4.5 Underactuated Systems: Defining and Counting Degrees of Freedom

How Many Independent Variables Describe a Configuration?

> Degrees of freedom (DOF): independent parameters specifying configuration—position and attitude—under the model and constraints.

System	DOF	Reason
Free Spatial Rigid Body	6	Three translations and three rotations
Rigid-Body Aircraft Model	6	No ground constraint; same as a free spatial rigid body
Ideal Planar Vehicle	3	Two positions and one heading
Multibody Mechanism	$6N - \sum f_i$	N rigid bodies have $6N$ freedoms; subtract the constraints at each connection

Degrees of freedom follow the model and geometric constraints, not the number of motors or control inputs.

1.4.5 Underactuated Systems: Multibody Degrees of Freedom

Multibody Degrees of Freedom: Free Motion Minus Independent Constraints

- Body count: Eq. (1.1), $6N - \sum f_i$. Vehicle plus three arm links: $N = 4$; before connection, $6N = 24$.
- Revolute joint: one relative rotation remains; $f_i = 5$ constrains three translations and two rotations.
- Floating base: three joints impose 15 independent constraints, leaving 9 freedoms: 6 body and 3 joint freedoms.
- Fixed body: a stand removes 6 body freedoms, leaving 3 joint freedoms. This example is an open chain.

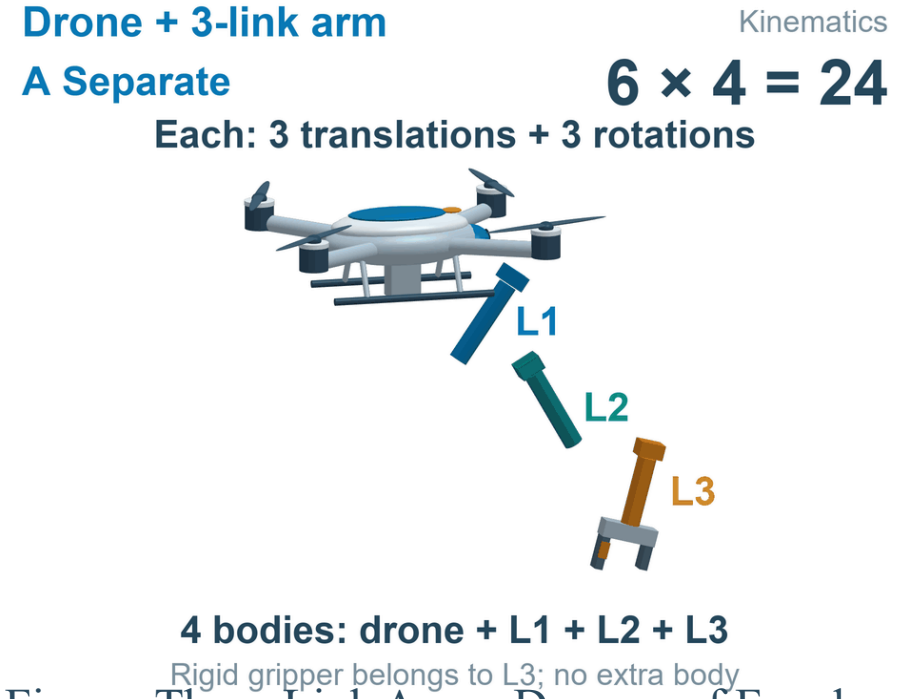


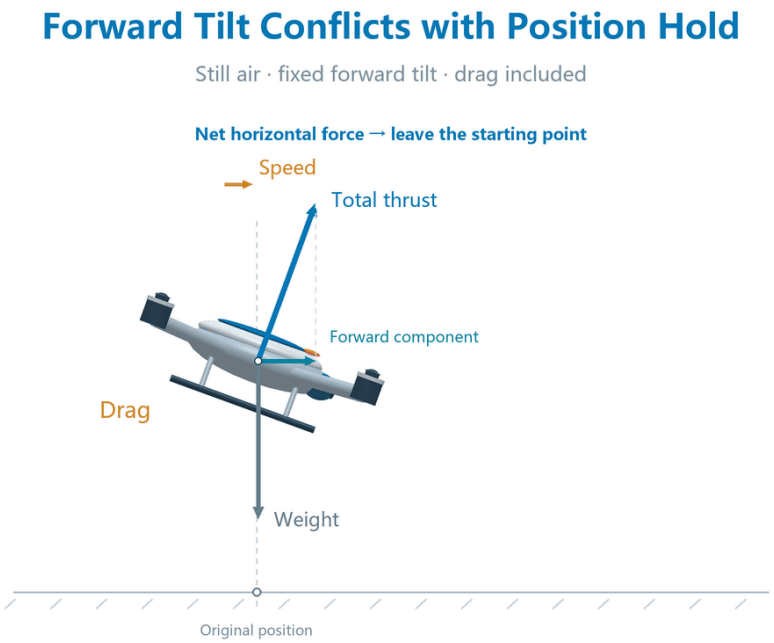
Figure: Three-Link Arm—Degrees of Freedom and Constraints

Bodies, base conditions, and independent constraints determine freedom; count duplicate constraints only once.

1.4.5 Underactuated Systems: Control Principles

Underactuation: Fewer Independent Inputs Than Degrees of Freedom

- **Definition:** a system is underactuated when it has fewer independent control inputs than degrees of freedom.
- **Conventional multicopter:** 6 freedoms but only 4 independent inputs—total thrust and 3 moments. Thrust always follows the rotor axes.
- **Direct consequence:** without horizontal external forces, hover requires a level attitude; horizontal acceleration requires tilt.
- **Further check:** even enough inputs must generate mutually independent forces and moments.



Drag limits speed, but cannot hold the original position

Underactuation limits simultaneous motion targets but still allows useful trajectories.

1.4.5 Underactuated Systems: Typical Constraints

Inputs and Contact Conditions Limit Instantaneous Motion

> These simplified models count independent inputs; equal counts can produce different independent forces and moments.

Typical System	DOF	Inputs	Motion Constraint
Conventional Multicopter	6	4	Cannot hover stationary while tilted without horizontal external force
Conventional Planar Vehicle	3	2	Cannot translate sideways instantly; must steer along a feasible path
Conventional Fixed-Wing Aircraft	6	About 4	Relies on relative airflow; cannot normally hover stationary

Actuation and contact constrain instantaneous motion and therefore feasible trajectories.

1.4.5 Underactuated Systems: Feasible Paths

A Planar Vehicle Can Reach the Side by Steering Along a Path

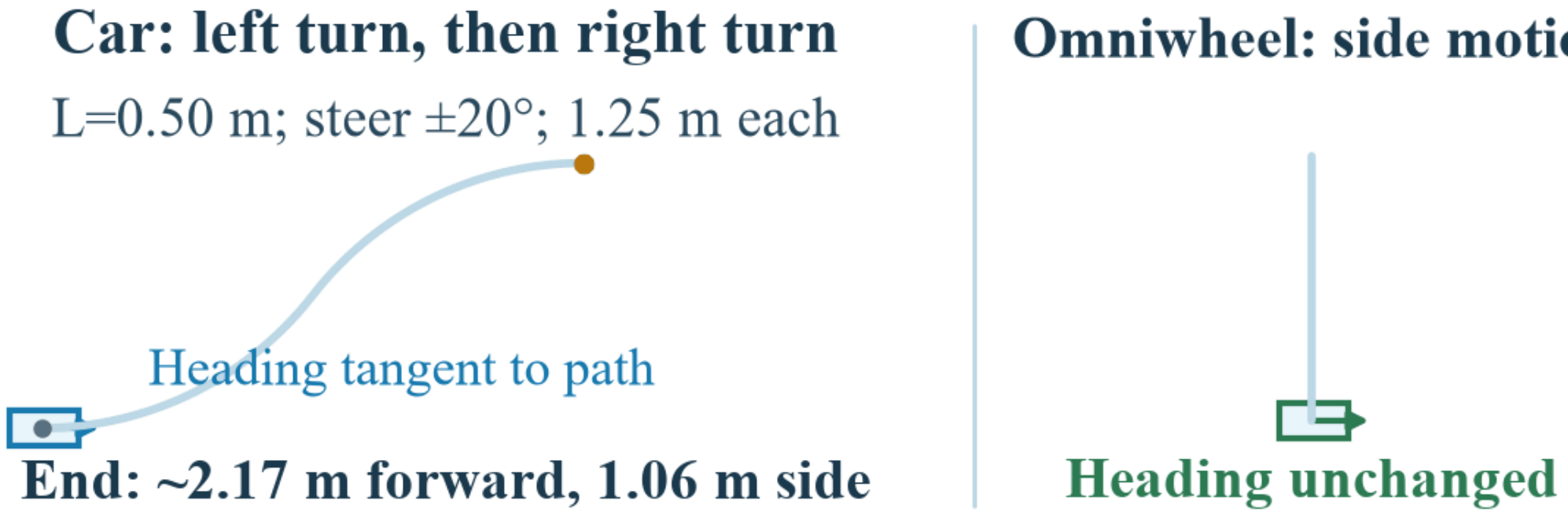


Figure: Conventional Steering Versus Omnidirectional Lateral Motion (Ideal Kinematic Simulation)

Inputs and contact constrain instantaneous velocity; steering paths can still change the final position.

1.4.5 Underactuated Systems: Command Combinations

"3＋1" Interfaces: Motion Targets and Motor Commands

> "3＋1" groups four high-level targets; position control sets roll and pitch from the required thrust direction.

High-Level Target Combination	Example Size	Purpose
Desired attitude ＋ total throttle	3＋1	Attitude target and thrust level
Desired velocity ＋ yaw rate	3＋1	Continuous motion and turning
Desired position ＋ yaw angle	3＋1	Waypoints and position hold

Compatible high-level targets let lower-level control coordinate attitude and thrust within actuator capability.

Class Discussion and Reference Answers

Connect Reference Frames, Forces, and Input Constraints

- Horizontal motion: why does a conventional multicopter usually pitch down to accelerate forward? Tilted thrust gives a horizontal component; fixed-wing propulsion provides forward thrust directly.
- Attitude singularity: ZYX Euler angles are singular at $\theta = \pm 90^\circ$ and sensitive nearby. Inverse solutions may jump when crossing; use quaternions or rotation matrices internally.
- Planar wheels: do four wheel speeds mean four freedoms? An ideal Mecanum chassis independently controls two translations and yaw, giving only 3 planar body freedoms.
- Order check: set $\psi = 90^\circ$, $\phi = 30^\circ$ and find the nose direction with column vectors. $R_z R_x$ gives $(0,1,0)^{\mathrm{T}}$; $R_x R_z$ gives approximately $(0,0.87,0.5)^{\mathrm{T}}$. In the former, roll leaves nose direction unchanged.

Motion predictions depend on the model, direction conventions, force relationships, and control capability.

Section Summary

Direction Conventions and Achievable Motion

- **Actuation and forces:** identify actuators, media, and moment arms, then analyze resultant forces and moments.
- **Coordinates and attitude:** specify position origins, vector reference frames, and attitude rotation order and positive directions.
- **Feedback and constraints:** the controller uses state to compute control demands; the allocator finds actuator commands within output capability.
- **Next:** Assembly and Debugging applies airframe, power, sensor, and actuator interface relationships to actual hardware.

Consistent coordinates, attitude, and force models give control allocation a clear physical meaning.